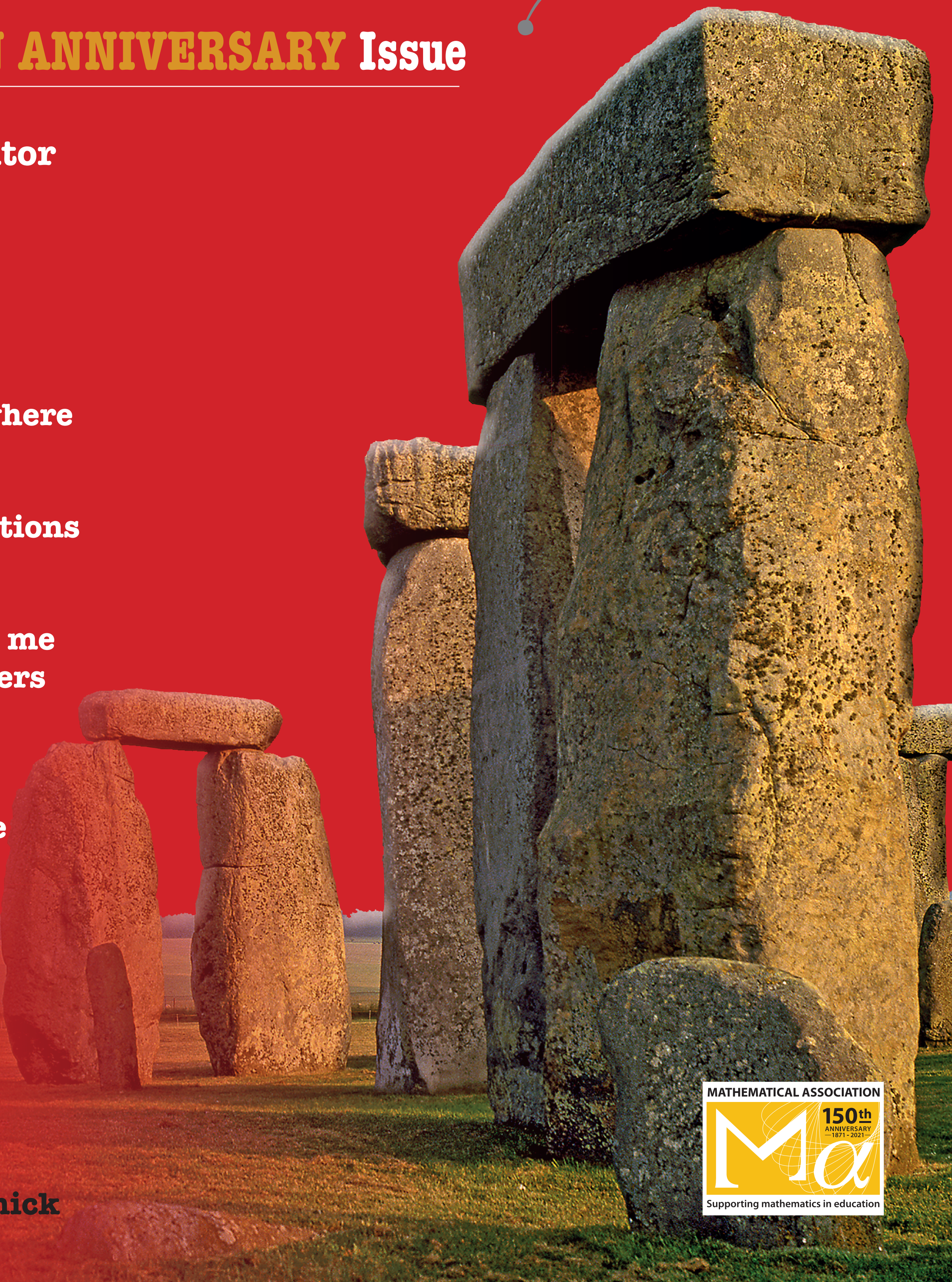
MATHEMATICS IN SCHOOL

NOVEMBER 2021 – Vol.50 – No.5

Celebrating 50 YEARS

The GOLDEN ANNIVERSARY Issue

with Guest Editor
Rob Eastaway



MATHEMATICAL ASSOCIATION
150th ANNIVERSARY 1871 - 2021
Supporting mathematics in education

MATHEMATICS ~~IN~~ *Outside* SCHOOL

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I felt honoured and delighted when I was asked to guest edit this issue of *Mathematics in School*, because this 'Golden Anniversary Issue' of the journal coincides with the 150th anniversary of the Mathematical Association's founding.

My thoughts then turned to what personal angle I might bring to 'my' edition. I am not, and have never been, a classroom maths teacher. I'm not well placed to pontificate on how to structure lessons or how to deliver the current stretching curriculum, and I'm certainly not qualified to suggest how a GCSE student can improve their grade from a 5 to a 6.

On the other hand, what I do have is a view from outside the maths classroom. In my work I deal with a broad mix of the public, from young parents to pensioners, and from theatre technicians to cricket administrators. All of them have an opinion about maths that they want to share – after all, as they frequently like to point out, they had to do maths at school too.

And it should come as no surprise that the overwhelming view from outside is that there is something wrong with the school maths experience. It seems that much of this is down to the excessive focus on exam results, at the expense of both the joy of the subject and the development of mathematical skills that will actually be 'useful' in later life.

The conversation in the maths education community can sometimes appear to be taking place in a bubble. I wanted to give a voice to adults who haven't been near a classroom for years, and also to students who are the intended beneficiaries/clients/victims (delete as applicable) of all of this. Hence my decision to rename this particular edition *Mathematics Outside School.*

As well as highlighting some of the concerns with the current system, I wanted those outside maths teaching to suggest solutions. How can maths be made more engaging for those who do not think of themselves as mathematical? What can be done to build stronger links between maths and other school subjects?

In putting together this edition I'm grateful to the regular editors for saying yes to (almost) everything. Thanks also to Gill Buque, Adriana Gonzalez, Richard Harris, Ben Sparks, David Lehmann, Mary Ellis and a couple of others who asked to remain anonymous. All have been incredibly helpful behind the scenes. Whether you are a maths teacher or a curious 'outsider', I hope you find this edition interesting and thought-provoking.

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Editorial Committee

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MATHS
ACROSS THE PROFESSIONS

by **Andrew Jeffrey**

In 2020, Rob Eastaway and I produced a short series of podcasts called *Puzzling Maths.* As part of the series, we invited guests from a range of interesting professions to tell us what part maths plays in their professional lives.

We had no prior knowledge of our guests' views or experiences of maths, so while this is not a large enough sample to be representative of the population at large, the guests do give a fascinating insight into how maths is viewed and utilized in different careers.

The interviews were wide-ranging, but for the edited versions below we have picked out each guest's answer to just three of the questions:

- What maths do you do in your job?
- How was maths for you at school?
- If you could teach maths for a week…?

As you will read from the transcripts, each guest gave their own unique slant on maths, but some common themes emerged. Most notably, guests frequently referenced the importance of making school maths feel relevant to everyday life and work, and how in some cases they were let down by the curriculum. Several also referred to how important studying statistics had turned out to be. (See Note 1 at the end of the article on page 4.)

ANNA FOSTER
BBC Radio 5Live presenter, journalist

What maths do you do in your job?

Maths crops up all the time in my job. Many of the stories we do involve maths, mostly when talking about money, but also statistics, without which we couldn't tell that story in a context that means something to our listeners. People, including politicians, always highlight the numbers that are most favourable to them. There's a responsibility on journalists to make sure those numbers make sense and also to make sure that, if only one figure is quoted, we put that in context or present other figures.

People often criticise journalists for joking that they are no good at maths. There's some truth in that accusation, but it's not that journalists are boasting about their poor maths, it's more an apology. As journalists we are expected to know everything about everything, but while you might be able to busk a knowledge of English or history, it's very hard to busk in maths the same way. And when I'm doing maths, I like to do it in peace so I can concentrate. That's not possible when I'm in the middle of broadcasting a radio programme.

How was maths for you at school?

I wouldn't describe myself as a maths person but I was in the top stream, I did GCSE in November of Year 11 (and got an A) and then did a stats GCSE in June with my other exams (and got a B). I never studied maths again because at the time I had a narrower view of what skills were involved in journalism. If I'd chosen to do A Level maths, I would have had to sacrifice something else. It would be nice if later in life there were opportunities to do a maths-related qualification that wasn't as all-consuming as an A level (see Note 2). It's only as an adult when making sense of a pay slip or working out a mortgage that you see the importance of the maths that you did or didn't pay attention to.

If you could teach maths for a week…?

I think that when you're school age you want to know what that real world context is. When might I need to calculate the circumference of a circle or use algebra in the real world? If I were to teach a maths lesson, I would like to explore its real world applications, following the example of people like Martin Lewis, the Money Saving Expert. He's spent a lot of time looking at the concepts of money and personal finances being taught to pupils in a way they will understand. Far too many of us as adults are mathematically illiterate about things we ought to know about.

SUE GREANEY
Archaeologist, English Heritage

What maths do you do in your job?

I'm a prehistorian and my job involves presenting information about archaeological sites like Stonehenge to the public. This means staying up-to-date with the latest archaeological thinking, which can involve complex statistics and data.

My favourite part of archaeology is that it's like a detective puzzle, combining your knowledge with data. You have to use the word 'probable' and 'likely' a lot in archaeology. So probability is a feature of my work too.

Maths is also involved in carbon dating, where we estimate the age of organisms based on the amount of radioactive carbon that we find.

How was maths for you at school?

I did maths GCSE and enjoyed it, but wasn't amazing at it. When I chose A levels, maths wasn't on my radar. I had other subjects I wanted to study. I knew I wanted to be an archaeologist so I picked the subjects linked to that: Geography, Biology, History. In Biology we did quite a lot of stats which was a good grounding for things I do in archaeology. I regret having once known things in maths, basic percentages for example, and got worse at them because I haven't studied them for years. If I'd carried on with maths it would have stuck a bit more. I remember at university that a lot of people struggled with maths because they were rusty.

If you could teach maths for a week…?

As an adult I really enjoy watching the histories of maths on TV. I wish somebody at school had told me why it's called algebra and who Pythagoras was; I had no idea these things were all from real people. I think knowing the history of these subjects would give people something more concrete to remember. Meanwhile history could learn from maths. It's about knowing what to believe, which sources are correct, what data we are seeing over time, correlation and causation. Knowing how to interpret statistics is so important in history and in the wider world, and that critical way of looking at things is great for historians to understand. At school I thought of maths as standing on its own not relating to anything else, and of course I now know that it's connected to everything.

SAMARA GINSBERG
Cellist and YouTube phenomenon

What maths do you do in your job?

I am not aware of consciously using maths in my music. I suppose that music is based on subdivisions of time and time is quantifiable, but I don't feel like I'm using maths skills when I think about rhythm. I often hear it said that there is a link between maths and music and that people tend to be good at both of them. My personal observation of professional musicians is that there seems to be the same 'Normal' distribution of maths aptitude and interest among musicians as there is in other people.

I'd say the maths I use in my adult life is almost entirely basic numeracy. I know people who were failed by maths at school. They are intelligent but have no concept of dealing with numbers. It's such a vital skill and if you don't have a handle on it you're going to get ripped off your whole life. The level of anxiety that people who think they're bad at maths have over this can be quite crippling. In my life as a freelancer I'm often talking to clients, they offer fees and I need to be able to say immediately if they are acceptable or not. If you don't have an instinct that a number feels wrong, you're in trouble.

How was maths for you at school?

Maths always came very easily to me. Unfortunately, being a girl at school who was good at maths was like walking around with a sign on your back saying 'kick me'. I was bullied so I set about trying to be bad at maths, escalating to the point of deliberately failing tests. I just wanted to fit in. I remember sitting in my Year 9 exam thinking 'how can I find a way to get these questions wrong without my teachers discovering I'm doing this deliberately'. If I'd put as much effort into trying to succeed as I did into trying to fail, I'd probably be a rocket scientist by now.

If you could teach maths for a week…?

I'd want to teach numeracy, but I'd also be happy to teach abstract topics such as how to solve quadratic equations. I don't think that learning in school has to be about application in the real world, it should also be about exercising and nurturing the developing brain. I don't think there's anything in school that has no point to it – it has an application to the development of a healthy intellect.

MIMI NWOSU
Civil Engineer, Sir Robert McAlpine

What maths do you do in your job?

It depends a lot on the assignment. Until recently I was working on the HS2 project and I was doing statistical analysis every day. When we were testing concrete to ensure it was meeting the designer's strength specifications, I was regularly using Excel to calculate means and standard deviations of the lengths of cracks.

How was maths for you at school?

Maths was never my favourite subject, but in Year 9 I had a maths teacher who really cared about my maths education, and because he showed interest in us I began to care about maths. What I particularly liked about him was that he showed us where maths was applied in real life. I can remember one lesson about Pythagoras, when he explained to us how it's a key part of creating and using maps, and I thought: "wow!". I started maths AS in sixth form but stopped after a week because I could see I was going to struggle, and swapped to Psychology. I didn't really enjoy doing maths until I started my Civil Engineering degree (see Note 3), when suddenly all the maths I was studying was directly relevant to what I was interested in.

If you could teach maths for a week...?

I would follow my teacher's example, and spend a lot of time talking about how maths links to careers – not just engineering, but games design, hair & beauty, electronics. I would invite external speakers to come in and give us talks. And I'd also want to teach financial maths, the economics of daily life, things like exchange rates and taxes. As an adult I can see how important maths is in a way I never realised when I was at school. I wish now that I'd paid more attention to maths earlier on in my secondary education.

SARAH LEPPARD
Optometrist and optician

What maths do you do in your job?

In my job I do everything from checking the health of people's eyes, to working out the power of the glasses that you need and fitting them. An optometrist's job is more physics meets biology than maths but the optician is more mathematical. Contact lens fitting requires a lot of working out as the power of an eye lens is very different from the power of a lens on your glasses. I use equations every day.

How was maths for you at school?

At school, I wasn't very good at maths and struggled. I took maths and statistics to GCSE and then dropped it. For me maths didn't resonate unless it made sense or fitted somewhere in my life, and it had context.

At A level I chose Physics and did abysmally and was told I should have taken maths A level. However if I had done A level maths, I fear I might have struggled just as much because it was so abstract. It wasn't until I did a foundation degree in Opthalmic Dispensing that I discovered the relevance of maths, and then I stuck with it. A context made it interesting and relevant.

If you could teach maths for a week...?

If I taught a maths lesson, I'd try to show kids how it is used in jobs, because there isn't a day goes past when maths doesn't come up in life. I remember sitting in Year 9 or 10 doing trigonometry and trying to get my head around sin, cos, tan, and just thinking 'this is just never going to be relevant in my life'. Years later I discovered that these concepts are really important in optometry! I knew I wanted to be an optometrist when I was eleven. I wish I'd known I'd need trigonometry for that.

Notes

1. To hear the full versions of these interviews, search for 'Puzzling Maths' on your preferred podcast provider.
2. In other words, she wishes she'd had a chance to study Core Maths (see page 5).
3 Most Civil Engineering degrees do require an A Level in maths. While at university, Mimi switched to Civil Engineering from the Science degree that she was studying.

Keywords: Relevance; Career; Statistics: Engineering.

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Where does the maths go?

by **Jude Mortimer**

A report published by National Numeracy in 2019 (*Building a Numerate Nation: Confidence, Belief and Skills*) showed that

> 'A quarter of adults have 'acceptable' levels of numeracy, with around half at the level expected of a primary school child.'

Sounds shocking right? These results came from a poll of 2000 adults aged 16-75 conducted in partnership with Ipsos MORI and included 5 questions, mostly percentage-related, such as:

1. If a scarf costs £11.70 after a 10% reduction, what was the original price?

a) **£12.50**
b) **£13.25**
c) **£13.99**
d) **£13.00**
e) **I don't know**

The report states that

> 'The results showed that 56% of respondents scored 2 or fewer (roughly equivalent to the level expected of a primary school child).'

So where does the maths go? Is secondary school maths a complete waste of time for most?

I teach maths (Functional Skills, GCSE and more recently Core Maths) at an adult college in South East London. My students are adults aged from 19 to 75 and from a wide variety of backgrounds. The one thing they all have in common is they need to pass a maths qualification. Does that make them 'bad' at maths? Have they forgotten it all? I would say the answer is no, at least not all, but they do bring their own real-life flavour of maths to the table.

When I was discussing ratio with a group of Level 2 Functional Skills students recently, they gave me their own examples of having used ratio. One used to work in a nursery and had to follow ratios to organise the correct numbers of adults to children, one is a paramedic and uses ratio to mix the right combinations of medication and another talked about mixing the correct ratios for hair colouring. The typical sweet-sharing questions seemed rather dry in comparison!

We progressed onto a good old traditional inverse proportion question.

> *It takes 3 people 7 days to build a fence.*
>
> *a. How long would it take 6 people to build the same fence? (1 mark)*
>
> *b. How long would it take 7 people to build 2 of these fences? (2 marks)*

The second part, in particular, flummoxed most of the students. I was reminded of a conversation I'd had with Ben (a GCSE student with dyslexia) a few years ago. He hadn't batted an eyelid at this question and had just told me that the job was worth 42 days and you needed to share that number between the 7 workers. Ben worked in construction and dealt with this kind of question every day. He probably would have happily costed it for me too! I was also delighted to find that Ben used the 3, 4, 5 rule (Pythagoras) to make sure angles were right-angles. Real-life maths at its finest!

Derek wanted to make a steel octagon to cover an unsightly circular sewer drain in his daughter's garden. The octagon had to measure 23.5 inches between opposite flat sides and he needed to calculate the length of each side and the angle to cut.

He described how he had split an octagon into 16 right-angled triangles. He calculated that each of these would have each a centre angle of 22.5° and used trigonometry (his explanation in the panel) to get the values he needed. The results speak for themselves!

23.5/2 = 11.75 inches (length of perpendicular from centre of flat to centre of Octagonal
11.75 × Tan(22.5) degrees × 2 = 9.73 inches (inside length of each Octagonal side)
90 – 22.5 degrees = 67.5 degrees (angle cut at each end of Octagonal side

I had a long (and eye-opening) discussion with Serena, a jewellery tutor, as to how she uses aspect ratio (internal diameter divided by wire thickness) to calculate the size of a jump ring used to make a chain. This not only gave her an aesthetically-pleasing result but meant that the beads placed inside, could move around but not fall out.

One of Serena's creations

That's not to mention Danielle, the speech & language therapist, who uses percentages to compare the number of tasks achieved by children, or Lorraine the former publican who is a whizz with VAT and finances or Jamie the carpenter who can convert between units of measure quicker than you can say 'dovetail joint'.

All here because they need a maths qualification but all brilliant in their own areas. It has really made me think about the relevance of maths in context. It has also made me question my definition of 'being numerate'.

Last summer I attended (online) some of the Core Maths Festival sessions (run by the Advanced Maths Support Programme) and was inspired! So much so that I persuaded management to let me start a small cohort.

For anyone not in the know, Core Maths is a Level 3 qualification aimed at students who want to continue their maths studies at 16 but don't want to take a full A-level. It is ALL about maths in context and covers topics such as data analysis, maths for personal finance, estimation, critical analysis and spreadsheets.

It's also been recognised as relevant for adults. In the summer of 2021, the government added it to the Lifetime Skills Guarantee list – courses funded for adults who don't already have a Level 3 qualification.

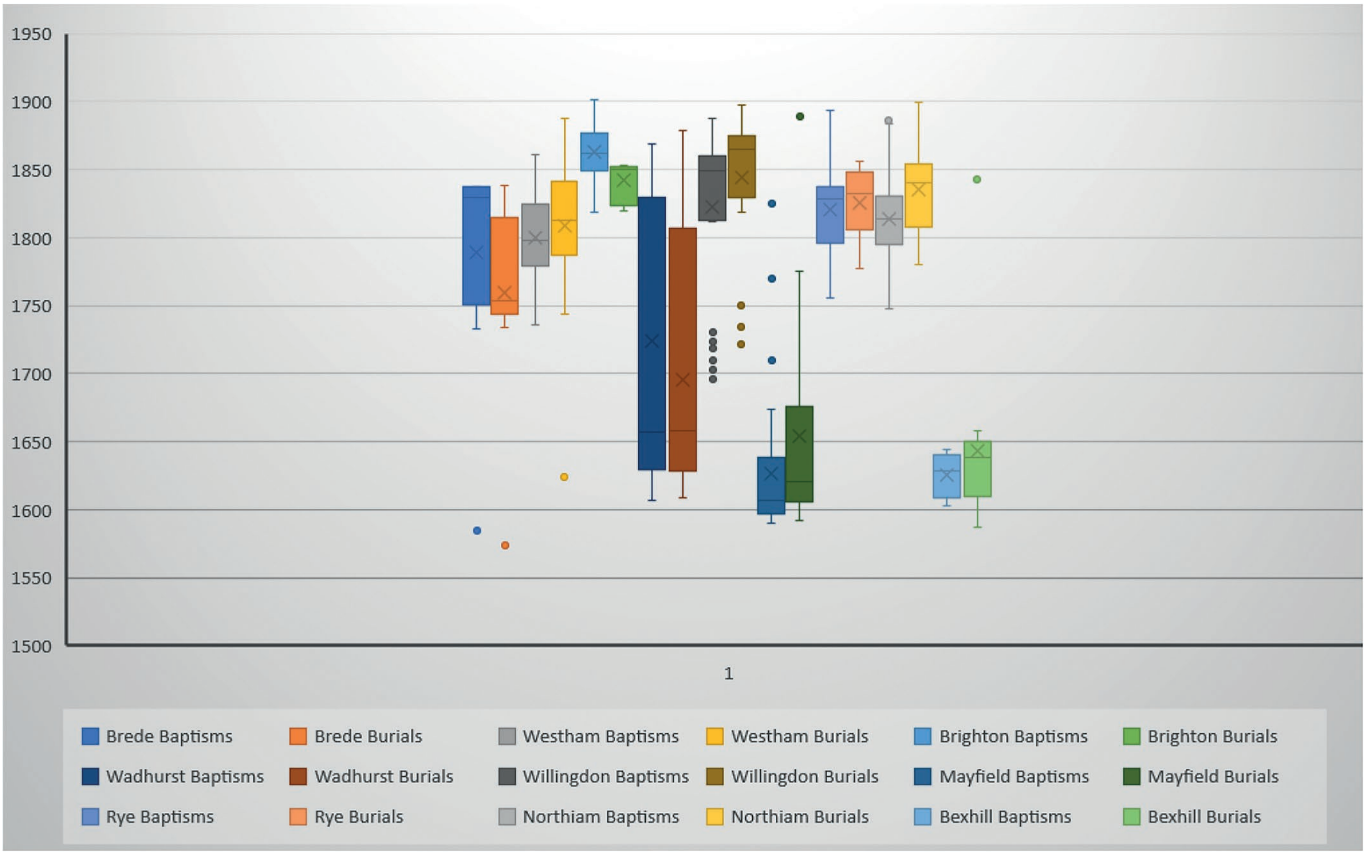
A Core Maths topic that has leapt out at me is described very neatly by one of my students:

> 'What has been a life changer for me is critical analysis and the vast world of the media and data. I now look at this with new eyes and a different mind, tending not to believe anything until I have looked at the data and/or analysed the facts for myself.'

In his book, *The Art of Statistics*, Sir David Spiegelhalter talks of the importance of 'data literacy, which describes the ability to not only carry out statistical analysis on real-world problems, but also to understand and critique any conclusions drawn by others on the basis of statistics.'

Critical analysis is maths we can all use every day, giving us the skills to examine newspaper headlines, statements

Core Maths students are encouraged to do a critical analysis of statistics in the news. Resources can be found at: https://amsp.org.uk/resource/where-maths-meets-the-world-of-work



A Core Maths student used Sussex Parish Baptism and Burial records to create this box plot, allowing him to visualise more clearly patterns in where his ancestors lived.

or claims, consider the narrative presented, make our own informed decisions, question rather than accept and have the confidence to use data to challenge and back up opinions.

To my mind now, maths is so much more than having knowledge or applying rules. For me, it is just as important for my students to have the ability to question, think critically, be curious. (What do you notice? What do you wonder?)

Given the right tools and inspiration, anything is possible. A Core Maths student surprised me the week after we spent some time studying box plots. He said,

‘On a personal level, the use of box plots, for example, in enabling the visualisation of thousands of family history parish records, enabled a plan of action that has solved problems I had been trying to solve for years. For me, a true Eureka moment.’

This is the maths that doesn’t get lost over time: the maths within a context, the maths we use in our everyday lives to help us make things easier, to answer questions and create new ones, the maths we use to help us make sense of the world around us.

So perhaps adults are more numerate than the National Numeracy report shows. It’s hard to measure such broad experiences in just a few questions.

In any case, it is clear how important real-life maths is, especially data literacy which Spiegelhalter says ‘is a key skill for the modern world’. As educators we want to provide our students with these skills and Core Maths is definitely a great step in that direction.

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Keywords: Core Maths; Numeracy.

Author: Jude Mortimer

Maths through a Parental Lens

by **Donald MacCormick**

When I told a maths-teacher friend of mine that I was writing an article for *Mathematics in School*, she was a little indignant: “You!” she said, “You may know about maths but what do you know about schools”. And she is right, my direct experience with Maths **in schools** is limited to my own school experience a number of decades ago (I will say O-levels and leave it at that) and, in the last couple of years, helping my son with his GCSE and A/S-level.

In fact, it was the experience of revisiting school maths with my son that was the driving force behind writing this article. I loved maths at school, I was always going to study it at university, I have re-read many of my textbooks for pleasure since graduating and I am a voracious consumer of maths on YouTube (particularly 3blue1brown). As you can probably tell, I needed no reason or encouragement to study maths, I just did it because I found it fascinating.

Even after leaving education, maths was part of my life as I joined a company specialising in mathematical modelling, thirty of us, all mathematicians of one sort or another bringing our maths to bear in areas as diverse as beer advertising, chocolate manufacturing, hotel occupancy, retail store layout and many more.

However, watching someone with a rather different perspective go through the GCSE exam process made me realise just how different my son’s experience was from mine. It wasn’t for lack of ability (he duly collected his grade 9 at the end of the process), it was rather that it was all a bit of a grind. There was no spark, no motivating reason, just something which had to be done to move to the next stage. It was also not for lack of enthusiasm on the part of his teachers, you could see that they tried to inject context, interest, and motivation. However, faced with pressure to maximise grades and a curriculum hell-bent on rewarding the mechanics of maths rather appreciating its wonder, it is not surprising that it became a game of mastering techniques to spot and apply in the exam then forgetting them afterwards.

This seems to be an unfortunate recurring theme in many of the articles in this issue of MiS. The questions keep coming up, “why am I doing this?”, “when will I ever use this again?”, “why is this so boring?”

So, what could be done? Better, more qualified people have put a lot of thought into this, so I doubt I will be able to make much difference, but if you will humour me, I would like to play “Fantasy Curriculum” for a minute or two.

It is ironic that maths gets so much criticism for not being relevant when we also often hear from the business world (and indeed from almost all of the interviews with professionals elsewhere in this issue) that we need more key basic numeracy and stats skills in our workforce. To fix this, my fantasy “compulsory maths skills” GCSE would probably consist of some combination of the material in:

1. *How to Read Numbers* by Tom and David Chivers
2. *Maths on the Back of an Envelope* by Rob Eastaway
3. One of the Core Maths syllabuses (e.g. AQA Certificate Level 3 Mathematical Studies).

I would do whatever it took to make sure that everyone was well versed and competent in these skills before they left school, even though that would inevitably limit their exposure to the more advanced topics which make up most of today’s GCSE curriculum.

I would partition the remainder of maths, wonderful things like surds, algebraic fractions, factoring quadratics, differential calculus, laws of indices, trigonometry, circle theorems, etc., as an optional subject, but one which channelled more of an “inner Attenborough”. Let me explain. When we sit down to watch Blue Planet or any other Attenborough documentary, we don’t start asking what relevance it has to our everyday lives, we just bask in the wonder and majesty of it. Why can’t we strive to show maths to school children in the same way? After all, if the subject is optional we are going to have to do something different because, as the curriculum stands today it would not be a popular choice. Let’s focus less on the mechanics and more on the beauty, the wonder, the fascination. Do we really want to present maths to children through pages of repeated surd simplifications which can be done by rote without any real understanding, and (I gather) sometimes reduce students to tears?

As I said earlier, much of this thinking was inspired by helping my son with his GCSE revision. One thing which keeps coming back to me from that is that they had to be able to convert recurring decimals to fractions. By the time of the exam he could do these standing on his head, an easy 2 or 3 marks as soon as he saw it. However, ask

him to do it today and I suspect he would just look at you blankly.

Contrast this with the idea that 0.9999 recurring is exactly actually equal to 1. He remembers that, not from GCSE but from a discussion he had with one of his primary school teachers who, ironically, refused to accept it was true. The fact that 0.999… is not just close to 1, but identical to it, is probably my favourite maths equality (I know, I should say Euler's identify and then be laughed at for being so hackneyed). 0.999… = 1, is an easy to articulate idea, which is counter-intuitive enough to be interesting and can be used to simply demonstrate the nature of proof and the nature of infinity AND maybe even lead into a discussion about Hilbert's hotel. What an opportunity to engage and enthuse.

Again, none of this is a criticism of teachers, I am sure something like this is done in classrooms up and down the country, but so long as the exams focus on mechanics rather than appreciation, it will inevitably be lost in the noise. Let's re-think how we teach maths, let's reduce the emphasis on mechanistic techniques that will be quickly forgotten and focus instead on things that will be remembered forever.

As my friend in the opening paragraph pointed out; who am I to be saying this with any hope that it stands up to scrutiny or stands a chance of coming to life? I would say the probability is about 1 - 0.999… Perhaps I should put my time where my mouth is and start a "Wonders of GCSE maths" YouTube channel, but for now, as someone who has a hard time understanding how people can look into the world of maths and not be filled with wonder and excitement, it feels better to have got it off my chest. Thank you for bearing with me.

Keywords: GCSE; Parents; curriculum.

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Double Measures

by **Rob Eastaway**

Few teenagers leave school with the ability to convert between metric and imperial units. After all, who needs to do that? As it happens it is a useful, and sometimes even essential, skill that will apply in a variety of professions. Here's a quick fix.

The nurse presents the baby girl back to her happy young parents.

"She's 3.2 kilograms," says the nurse.

"Er...could you tell us what that is in pounds?" ask the parents, awkwardly.

This is not a scene from *Call the Midwife*, it's a conversation being played out in maternity units across the UK every day. Those young British parents were educated in the UK, and their education was exclusively in metric units. "Stones" and "pounds" never once cropped up in their school lessons. Yet when it comes to babies – and the weights of other people, come to that – this couple think in imperial units. Why? Because they inherited this language from their families and from the media.

* * * * *

"How tall are you?", I asked my daughter.
"Five foot three, I think," she replied.
"What's that in centimetres?"
"I'm not sure."
"What about this kitchen that we're standing in, how wide would you say it is?"
"About… four metres, maybe?"
"How many feet?"
"Erm . I'm not sure."
Like many teenagers, she thinks of people's height in feet, but other distances in metres.

A roll of carpet for sale in a South London shop in 2021. The length has been measured in feet, the width in metres. And customers are supposed to make sense of this?

* * * * *

"BRITAIN SIZZLES IN 100 DEGREES", shouted the headline of the tabloid newspaper. Lest there's any confusion, the paper was referring to degrees Fahrenheit – which is interesting because six months earlier that same newspaper carried the headline:

"BRITAIN'S BIG FREEZE!
SUB-ZERO TEMPERATURES ALL WEEK".

Sub-zero? That's zero degrees *Celsius*, the freezing point of water. How strange to use one scale for hot temperatures and another scale for cold temperatures. Yet that newspaper was simply reflecting the way that a surprising number of British adults think: Fahrenheit for hot (because 100 is a nice memorable number) and Celsius for cold (because why on earth should 32°F be 'freezing'?) There's a middle temperature of about 10°C/50°F where many adults switch across from one measure to the other. I'm one of them.

* * * * *

Britain's units of measurement are all over the place. Our education system has been purely metric (aside from miles) since 1970s, yet most of us use imperial units as our benchmark for something in our lives, be that weight, height, or the volume of a unit of milk. But we're all different. *You* might think of weight in kg and your height in feet/inches, but your *neighbour* might well be a stones-and-centimetres person.

If you want to be generous, you could say that our charmingly eccentric mix of imperial and metric units is part of what makes us British, and in any case, it sort of works, doesn't it?

Well, *does* our mish-mash system work? I'm not sure. While people might be confident of particular units in specific situations, perhaps knowing somebody's height in feet is more like a 'label' rather than part of an integrated numerical system of measurement.

As an exercise in estimation, I like to ask Year 10s to guess how far it is from London to New York. The majority of their estimates are somewhere in the range 1,000 to 10,000 miles. The correct figure is around 3,500 miles, so their estimates aren't bad. At least they've got the right order of magnitude. What's alarming is when I then ask them to convert those miles to kilometres. Remarkably few are familiar with the rule of thumb that to convert from miles to km, you should divide by 5 and multiply by 8. But many of their estimates aren't even in the right ballpark. Here are some estimates that students made in miles, and their 'conversions' to kilometres:

Estimate in miles	**Conversion to km**
1,200	12,000
5,000	50,000
20,000	2,000
856,000	85.6

Answers like these were not uncommon. I suppose the students were used to measurement questions involving multiplication or division by powers of 10, and that was certainly the instinct here. Quite what they make of a road sign that says Paris is 50 km away, I don't know.

Accurate conversions

When Britain first went metric, there was a concerted effort to teach the public how to convert the old measurements they were familiar with. Kellogg's joined in. The following mnemonics appeared on the backs of Cornflake packets for several months:

Two and a quarter pounds of jam
Weighs about a kilogram
A litre of water's
A pint and three quarters
A metre measures three foot three
It's longer than a yard you see.

These conversions are quite accurate: A kilogram is a bit less than 2¼ pounds, more like 2.20. A litre is 1.76 pints, not 1.75. A metre is a tad longer than 3 ft 3 in.

Here's a table showing the handy conversions from and to the most common imperial measurements.

Units	Handy conversion	
	Decimal	**Fraction**
miles to km	× 1.6	× 8/5
inches to cm	× 2.5	× 5/2
litres to pints	× 1.75	× 7/4
kilograms to pounds	× 2.20	× 11/5
metres per second to mph	× 2.25	× 9/4
degrees C to degrees F	× 1.8 + 32	× 9/5 + 32
metres to yards	× 1.1	× 11/10

Many readers of more mature years will not only be familiar with these conversions, but might even be adept at working some of them out in their heads. 65 miles? $65/5 = 13$, and $13 \times 8 = 104$ km. But while the ability to convert accurately is a nifty party trick, it could hardly be called an essential life skill, and not many school students would be motivated to learn how to do it.

However, there is a skill that every student should be encouraged to learn: the ability to double (and halve) mentally. Why? Because if you want a ROUGH conversion from imperial units to metric units, then doubling and halving will give (or lead to) a decent answer:

Units	Rough conversion
miles to km	Double
inches to cm	Double
litres to pints	Double
kilograms to pounds	Double
metres per second to mph	Double
degrees C to degrees F	Double and add 30
metres to yards	They are the same

A quick-fire exercise in class would help students to become more familiar with measurement units. Since the conversions are rough, I suggest that they round their answers, and use the ending 'ish'.

What's 15 litres in pints?	*Double 15 ... 30-ish*
What's 9 inches in cm?	*Double 9 ... 20-ish*
What's 70 mph in metres/sec?	*Halve 70 ... 35-ish*
What's 84 pounds in kg?	*Halve 84 ... 40-ish*
What's 100 Fahrenheit in Celsius?	*Take away 30 then halve ... 35-ish*
What's 22 yards in metres?	*The same ... 20-ish*

One thing they will have to remember, of course, is whether the conversion involves doubling or halving. But it's an important life skill to know that a litre is bigger than a pint.

If your students leave school able to do conversions like these, they will be able to do something that at least 95% of their peers can't do. And their future employers will be surprised and, quite possibly, impressed. Meanwhile check out the classroom game in the panel below.

DOUBLE OR QUIT – a classroom game

Here's a quick game you can play to embed the skill of mentally doubling numbers. Start by picking a random whole number between 1 and 1,000 (it could be drawn from a hat, or generated by a calculator). Pick a student to begin the game. Their challenge is to double the number you started with. And then to double it again, and to keep doubling until they are no longer confident that they will get the right answer. When they choose to 'quit' the number they've reached is their score. Another student can then volunteer to pick up the baton and double the number that the first student reached, and keep doubling until they quit. Any error in the answer, however, and that player loses all their points. You can keep a leader-board of the highest scores reached this week. And of course, a student who's weak at arithmetic might get lucky – the random number they are given to start with might be 967, at which point they can quit on a high score.

Keywords: Estimation; Conversion; Metric system; Imperial system.

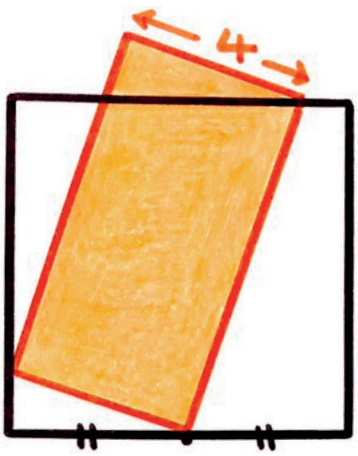
Author: Rob Eastaway, contact details on p.1

GEOMETRY PROBLEM 6 : SOLUTIONS

Collated by **Chris Pritchard**

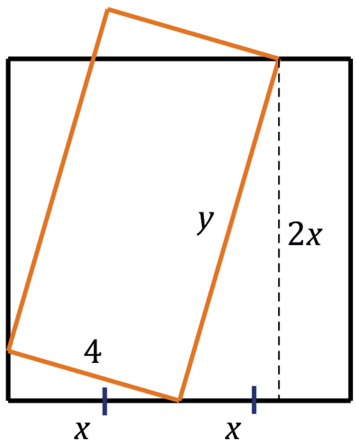
In the May issue (Vol. 50, no. 3), we presented the sixth of Catriona Shearer's original geometry problems and asked for solutions to be sent in. Here's a reminder of the problem.

An orange rectangle and a black square. What's the shaded area?



Near-identical solutions were received from Gerry Leversha, Howard Fay, Peter Swindale, Chris Dean, Graham Wills, Geoff Strickland, Floriana Pacchiarini and Nick Bowley, and a similar one using trigonometry from Belinda Gardiner and again from Floriana Pacchiarini.

Gerry let the side of the square have length $2x$ and the longer side of the rectangle, y.



Then he dropped a perpendicular from the upper right vertex of the rectangle onto the base of the square (dashed).

By similar triangles,

$$\frac{x}{4} = \frac{2x}{y}$$

and so $y = 8$ and the area of the rectangle is 32.

Gerry also commented that 'this would work even if the rectangle spilled over the right-hand edge of the square'.

In addition to solving the original problem, Peter Swindale decided to seek the length of the side of the square, or at least to examine two limiting cases. Firstly, when the upper right vertices of the square and rectangle coincide, he noted that $5x^2 = 64$ and hence $2x = 16/\sqrt{5}$. And trivially, when the rectangle is upright, $2x = 8$. He further speculated that the distance between the upper right vertices of the shapes is about $x / 3$, which leads to:

$$2x = \frac{12\sqrt{10}}{5}.$$

Incidentally, if you happened to misinterpret the width of the orange rectangle as IX (i.e. Roman 9), then the area of the rectangle is LXXII. Yes, it did happen ... but no names!

And I can add a little more colour to this round-up with the following two stories.

(1) Patricia King sent me a rather long-winded (but correct) solution involving Pythagoras' Theorem. Meanwhile, her 15-year old son, Isaac, showed her the similar triangles solution, perhaps to her chagrin but more likely to her pride.

(2) Graham Wills's solution arrived at MA headquarters by postcard, and belatedly a similar postcard arrived with a correct solution to Problem 3. Graham is a very longstanding member of The Mathematical Association. His cards confirm that by publishing Catriona's geometry puzzles in *Mathematics in School* they are reaching a new audience.

It is good to see the number of submitted solutions increasing with every problem posed, and to see families and twitterophobes engaging with them.

Keywords: Geometry; Similar trangles.

Author: Dr Chris Pritchard, contact details on p 1

Maths GCSE and Me – **What Sixth Formers Think**

by **Nicole Cozens**

Elsewhere in this edition you'll find plenty of views on school maths from professionals, parents, former teachers, and practising mathematicians. But there's another group that have an interesting perspective on school maths: those who took GCSE maths in the last couple of years, and for whom the maths classroom is still fresh in the memory.

We contacted a variety of schools across the country, inviting their sixth formers to reflect on their experience of GCSE maths. We asked them to submit 500 words, telling us what they enjoyed, and what suggestions they might have about the curriculum and the teaching.

We received numerous essays from sixth formers. At one extreme were those who have given up maths completely and are concentrating on the Humanities; at the other were those for whom maths on its own is not enough, and who are studying Further Maths and Physics as well. The GCSE grades of our respondents ranged from 4 up to 9. Yet despite this wide range of maths achievement, there was a surprising consensus in the students' experiences and opinions of GCSE maths

One thing that stood out was that students often identified the power of a positive relationship between the teacher and student.

Kira Haket from Saffron Walden County High said:

> *My maths lessons were enjoyable and one of my favourite lessons to attend. I put this down to my teacher. He was always friendly and enthusiastic, making him one of the most approachable teachers in my secondary school (just like my A-level maths teachers). This made his lessons instantly more fun and meant that I felt comfortable approaching him for help if I didn't understand something, which all students need, particularly for maths.*

Luca Veronese commented

> *The biggest shock of my GCSE years was when I was told I was good at Maths. Up until the age of 14 I lived in Singapore where, thanks to their pressure-cooker approach to teaching, I was considered an average-to-low ability student, leading to a huge sense of self-doubt. So when, in a Year 10 parent-teacher meeting in the UK, Ms Routledge declared that I was good at maths, I was stunned. No one had ever said that to me before. It is one of my greatest regrets that I never thanked her for it.*

It's worth noting that Luca went on to get a grade 8 at GCSE.

Some students told us they got enjoyment from the logical structure of maths. As **Emma Kenny** of Loreto Grammar School put it:

> *There is something so satisfying about having a page full of working, and finally getting the answer perfectly.*

Most of the students understood the importance of succeeding in GCSE Maths and recognised how mathematics pervades many aspects of life outside the classroom. However, many also echoed the thoughts of **Oliver Haley** of Hills Road College in Cambridge**,** who felt that as he left his Key Stage 3 maths behind and approached his GCSEs, maths became little more than training to pass an exam:

> *QUICK! Learn this for our test next week and it WILL come up in your exam in June so REVISE THIS NOW AND LEARN IT OR YOU WILL FAIL!*

and whose maths teacher when asked by a student

> *"But Sir? Why do we need to know this" would often reply "You need to learn it for your exam, that's it."*

For many, the joy of maths disappeared as GCSE approached. **Miriam Waters** (who achieved Grade 9), told us that

> *I wasn't bad at maths, but I was fundamentally uninterested in it. As a creative student, maths seemed deeply rigid and, quite frankly, dull. I don't hate maths – I can admire the beauty of an algebraic equation and marvel at quantum physics, and I love the idea of Chaos Theory, though I don't understand it. But something about the attitude of both maths students and teachers put me off at GCSE.*

Yuki Azawa from Exeter Maths School (but who studied GCSE elsewhere) felt that

> *If you ask anyone in secondary school what lesson they dislike the most, the answer will probably be Maths – and the reason is that it is dull, grindy, and worst of all, uninteresting. What I mean by 'grindy' is that there is a lot of just bashing through the method which you've done countless times in countless past papers.*

I found reading the students' submissions challenging at times, as the essays didn't reflect my emotions towards

maths, or what I hoped students would feel. However, what I enjoyed was the solutions the students suggested for overcoming their negative experience of learning mathematics.

They revolved around two themes. The first was the importance of making maths relevant, putting the topics in context where possible, though this context has to be realistic. **Miriam Waters** wanted teachers

> *to show how maths might figure in, say, archaeology, or art history, or English. All the subjects cross over in some way, and our arts teachers always tried to interest the more STEM-focused students by explaining this. There was none of this in maths; It was as if they considered the rest of the world superfluous.*

Maya D'Angelico explained:

> *I now study Art History, fortunately, there's no Maths involved in the A Level course. However, I've noticed that Maths does play a part in Art History, especially when it comes to authenticating artwork, it's something I wish I could understand and explore, not immediately tense up about as soon as I see any hint of it in an Art History documentary.*

Where a 'real world' context **was** used in maths examples, students felt that it was often forced and irrelevant. **Luca Veronese** said:

> *There is no point asking questions about fruit purchased in pounds in a cashless and automated society.*

The second theme (which tended to come through from those who got a high GCSE grade) was that students wanted more opportunity to develop their problem solving skills.

Yuki Azawa told us that

> *geometry questions were 'enjoyable' [...] in the way that they would make me think about how I work it out – it made me use my 'problem solving' brain.*

He also felt that GCSE should have more 10-mark questions for 'the people aiming for top marks who want, and deserve, to be stretched'.

Ben Folland said that he

> *found a satisfaction in solving mathematical puzzles*, acknowledging that "*maths is a creative subject*" and *Nobody finds solving quadratic after quadratic fun – it feels like you are a robot following an algorithm to do a useless task. This is not what maths is. But is it what the curriculum portrays it as?*

How do we reflect on the submissions from the students? It can be easy to read the essays and become defensive, to think that it's not like that in my classroom/school/course. Or to dismiss the students as a self-selecting sample, and not representative of the cohort. However, their views do probably articulate the views of many, and we can learn from these students.

It is clear that students value variety, challenge and relevance. We could argue to try to reintroduce this through coursework, or mathematical portfolios. But it's worth bearing in mind that the current curriculum is not that different from the maths we teachers studied ourselves, and yet we emerged with a passion for our subject. Why? Because we have experienced the creativity, frustration and beauty that it can hold. We need to ensure that our love for maths shines through, in the activities we ask the students to do and the words that we say. By doing so we can motivate the students to persevere when faced with the inevitable repetition that forms part of exam preparation, and to see that as a necessary part of developing as a mathematician, but importantly, not the only part. Embedding the wonder of maths throughout the curriculum, using problem solving and contextualisation can make maths more fascinating and challenging for students.

Perhaps then we can reduce the number of students who leave GCSE feeling that maths was not a subject that they connected with. Students such as Molly Reid from Hills Road Sixth Form College, to whom I will leave the last word. Her poem is on the next page.

Keywords: Views; Relevance.

Author: Nicole Cozens

Maths GCSE and me

by **Molly Reid**

Hills Road College, Cambridge – A-levels: English, History, Theatre Studies
(Molly achieved Grade 7 in GCSE Maths)

It's like,

We're in big school now
Big maths
Wanna know how
A triangle can give an equation
Begin putting ABC instead of 123
Working out how bad the fall would be
If Kevin fell off that tree
And somehow Megan
Has 53 lemons
And a cinema has a special deal
Buy 2 tickets
And get a free meal

Then I'm working out the square root of 181
These lines muck up my brain
And everything I learnt comes undone
Why do I have to solve this question in 1 minute
The pressure is too much
I end up breaking into a cold sweat
But I'm only in year 9
And maths isn't optional
Let's watch my confidence decline
Put in set 1 for year ten
Comprehending nothing every lesson
I can't do this again

But
At least my teacher was nice
Took her time
Gave me some great advice
We started swapping out triangles for
A weird shaped aquarium
And a tiny little museum
And maths became fun again
Yes, it was hard
But I started to gain
Confidence
I could do maths
The stuff I saw smart people do on telly
When I was a smelly
Little toddler
I could tell that 3-year-old girl
Maths is a whirl

Then came year 11
The numbers got longer
Counting 1.35297
Mocks were coming up
Suddenly I needed to know 60 different methods
Let's back up
Here
I thought maths was fun.
Now scrambling to find meaning
To these empty words
"find x when y = 3"
All my walls crumbled
It was just this silly little test
And me

So, I failed
What a surprise
I tried not to let it get to me
But the lessons dragged on
As I fell behind
Everything I learnt
Fell out of my mind
And once you've passed the point of catching up
Maths becomes a nag
And I'm trailing behind
Just my teachers drag
Course I know they are just busy
Can't keep track of us all
Wish I could just install
A calculator in my brain
Instead of crying over maths again
I should be happy
But words are more my friend
I could spend
24 hours straight
Writing about a simple plate
And still not get bored
Words just
Strike the right chord
Play a beautiful melody
That gives my mind clarity
I wish we could just play to our strengths
Let predestination decide
Where we end up
Listen to how we feel inside
Yes maths is great
For the people who understand
But if you demand
Everyone to think the same
Of course, we will complain
My brain likes words
Your brain likes numbers
Let's work together
Instead of trying to
Measure each other
On our weaknesses
Maths can stay in the brains of those who like it
And I will use a calculator

Extracurricular Maths Sports

by **Fiona Yardley**

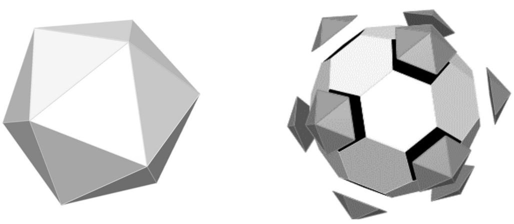
Elsewhere in this special edition, we read about the diverse ways in which mathematics is used by archaeologists, journalists, musicians and opticians and how it is, in Sue Greaney's words, 'connected to everything' (page 3).

As a historian turned mathematician with a knitting obsession, I am always excited by the opportunity to go "off curriculum", and at one time ran a club for able and keen year 5 and 6 students called 'Awesome Maths'. Each half term had a theme which allowed broad and varied exploration of mathematics (for example, mazes, art, time, optical illusions and maps). One half term was dedicated to exploring the many different ways in which maths is connected to sport.

Here are some of the sport activities that went down particularly well with both me and the young mathematicians.

Balls

Why is a football a truncated icosahedron? Starting from regular tilings, we used the patterns and generalisations we had noticed about angles in the plane to prove that there are only five regular solids. We built them and had some fun establishing that none of them was suitable for use as a football, which led us to see how the icosahedron can be adapted through truncation to a better approximation of a sphere.



Icosahedron and truncated icosahedron (football)
Images created by Ben Sparks

We discussed why we couldn't just have a sphere (because we needed a net constructed from 2-dimensional shapes) which led into a further session trying to construct nets for other sports balls (tennis balls, rugby balls, basketballs) and researching design and construction techniques for others. While we didn't go into the mechanics behind the dimples on a golf ball, there was no little excitement that it was mathematics which determined their existence and design.

Scoring

What final scores are impossible in rugby union? What final scores can be achieved in more than one way, and is there a pattern to this? Only two of the children knew anything about rugby before we started – another insisted that he had no interest in sport because he was a maths geek. Needless to say, by the time we had indulged in a little number theory exploring the different combinations of scores, he was a convert!

Subjective scoring created even more excitement. We started with the Salt Lake City skating scandal of 2002 and the consequent revamped scoring system which we agreed was still not perfect and so set about designing and then testing our own scoring system against a range of scenarios. Yes, not only were we doing maths in the context of sport, but we got real and passionate debate raging too!

Tournaments

In a knock-out tournament with 120 entrants, how many matches are played? The simplicity of the solution (always $n - 1$, because everyone loses exactly one match except the winner) wowed the pupils, as much as the complexity of tournament design that we moved onto next. We modelled tournaments based on a knock-out, round robin or hybrid (round robin followed by knock-out, as the football world cup) structures before discussing the relative pros and cons. Again, there was excellent debate exploring how mathematical models influence ideas of fairness and entertainment value. We also explored league tables, including having a beetle drive to critique ladder leagues and the history of how the football league tables have been decided and why we have our current system.

Keywords: Extracurricular; Sport

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GUEST EXAM **QUESTIONS**

by **Rob Eastaway**

Adults often comment that the maths they experienced at school seemed to have little connection to other subjects they were studying. What can be done to integrate maths with other subjects, and not just the STEM subjects, but the humanities, too?

Imagine if the end of year maths exams for (say) Year 10 had to include a Guest Question from a teacher in another department. For example the maths exam might have a guest question from, say, a History or an English teacher. This could work both ways, so the History or English exam could have a guest question from the Maths teacher.

The appeal of this idea is that it would make it clear to students that maths is a subject that is connected to every other discipline, and it would also force more cross-curriculum collaboration. Teachers who don't specialise in maths might also bring a novel and creative angle to the subject.

I asked teachers from a variety of departments at several schools to suggest maths questions that they might pose to a Year 10 class. The main criteria were that the question should not require specialist knowledge (but general knowledge is fine), and that it should be on a topic of interest to the teacher, not just maths-for-maths-sake.

I would like to thank staff at Long Road College Cambridge, Queen Elizabeth School in Bath and Westminster School for indulging me in this experiment. I've collected the questions here into what is perhaps the world's most eclectic – and eccentric – maths exam. The names of the teachers are fictitious as the content had input from several people, but there's something appealing about an exam question being personalised by naming the teacher who submitted it.

This is, of course, a thought experiment, not a formal exam. I can quite imagine that without preparation, many students would be completely thrown by some of these questions, and the mark schemes are arbitrary. Sometimes a 'one mark' question is worthy of a half hour discussion. So don't take this mock exam too literally. It's the principle of enriching maths by deliberately bringing in material from other subjects that is worth exploring further.

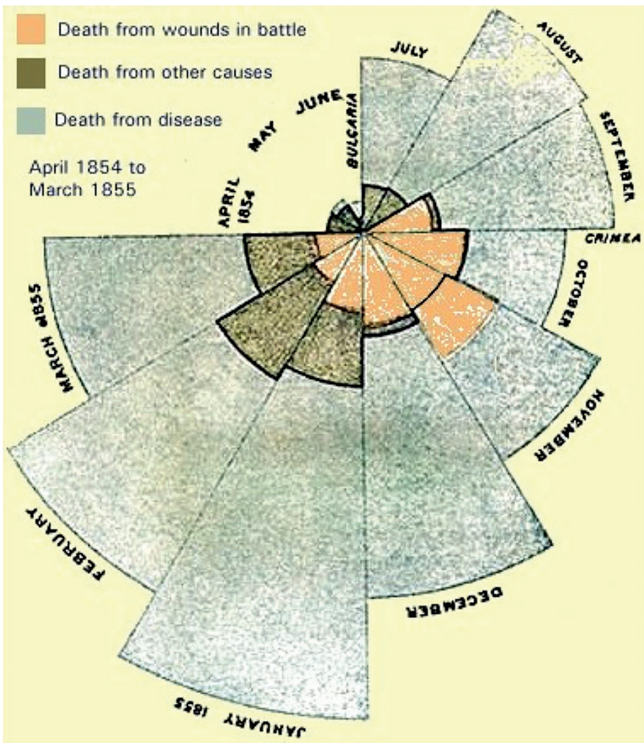
Keywords: Maths in other subjects.

Author: Rob Eastaway: contact details on p.1.

From Dr Tarvin, History teacher

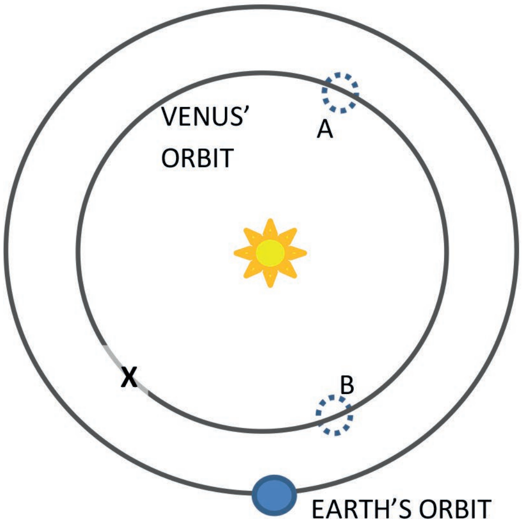
The 'rose' chart alongside was produced by Florence Nightingale in 1855, to illustrate the cause of deaths of soldiers during the Crimean War.

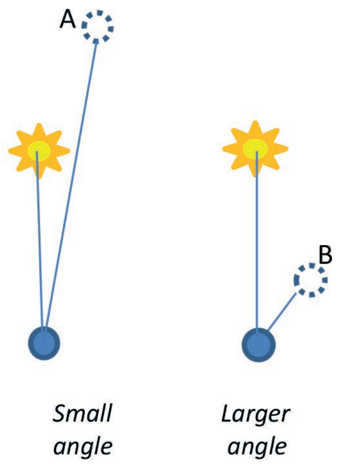
(a) In which month were there the most deaths from disease? (1 mark)
(b) What *two* important messages was Florence Nightingale able to convey with this diagram, about the priorities for health care? (2 marks)
(c) Using the chart, suggest which three months saw the most fighting. Explain your reasoning, and comment on what might be wrong with the colouring of the November 1854 segment (3 marks)
(d) What needs to be added to this chart if you want to know how many people died from different causes each month? (1 mark)



From Mr Kelsall, Physics teacher

Venus and Earth have roughly circular orbits around the Sun, but Venus is closer to the Sun than earth, as shown in the illustration. Imagine staring up at the sky to observe Venus. When Venus is in position A, the planet appears to be very close to the sun, because the angle between Venus and the Sun is very small. When Venus is at B it seems to be further from the sun because the angle is larger.





The point labelled *X* is the position in the orbit of Venus when the angle between Venus and the sun (as viewed from Earth) is at a **maximum.**

(a) What geometrical word is used to describe the straight line that passes through Earth and X?
T_ _ _ _ _ _ (1 mark)

(b) Draw a triangle joining Earth, the Sun and the point X. In this triangle, state the angle between Earth and the Sun as seen from Venus. (1 mark)

(c) Over several months, an amateur astronomer records the angle between Venus and the setting Sun. The astronomer notices that the angle sometimes gets very close to 45 degrees but is never higher than that. Use Pythagoras to show that the radius of the Earth's orbit is roughly $\sqrt{2} \times r$ where *r* represents the radius of the orbit of Venus.(3 marks)

From Miss Ashmir, Psychology teacher

A primary school teacher asks her class to think of their own secret number. Then she asks them to:

… double their secret number…
… then add 10 …
… divide by 2 …
… take away the secret number they first thought of.

The teacher announces: 'Your answer is the number … FIVE !!!' The children gasp in astonishment.

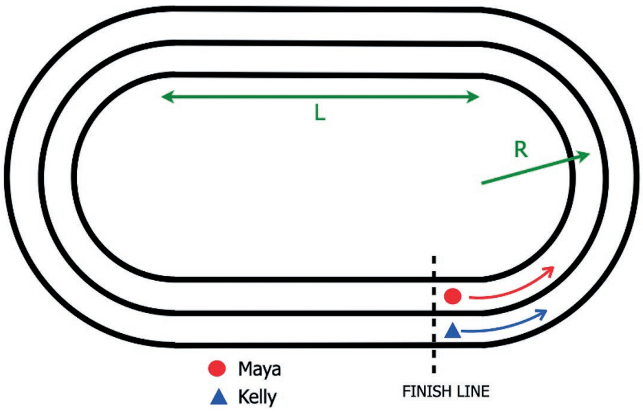
(a) Prove why the answer is 5 for any secret number 'N' that a child might think of. (3 marks)

(b) Why do you think the children are so surprised? (2 marks)

(c) Think of an example of when something in maths has surprised you. How important is surprise in arousing human curiosity? (3 marks)

From Mr Norley, PE teacher

An Olympics athletics track is divided into lanes, with two long straight sections, and semicircles at each end. Each lane on the track is approximately one metre wide. As they enter the final lap of the 1500 metre race, the front runner Maya is running in the middle of the inside lane as indicated on the diagram. Kelly is in last place and is running in the middle of the **second** lane. In order to overtake all the other runners, who are running in the inside lane behind Maya, Kelly has to stay in the middle of the **second** lane for the entire final lap.

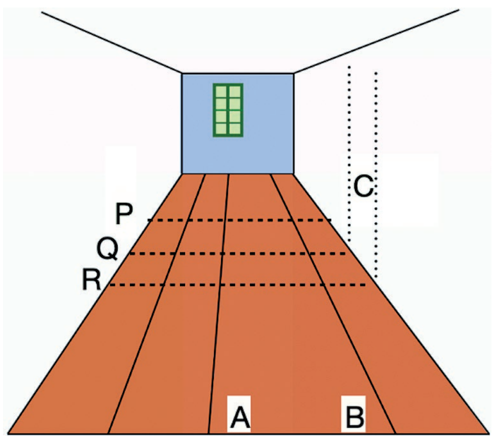


(a) If the length of the straight sections of the track is L metres, and the radius of the semicircle to the middle of the inside lane is R metres, write down an expression for how far Maya runs in the final lap. (2 marks)

(b) Estimate, in metres, how much further Kelly will run than Maya in the final lap. (2 marks)

From Ms Oakmere, Art & Design teacher

The illustration shows a view of a bedroom similar to the one painted by Vincent Van Gogh in Arles. There are wooden floorboards running along the length of the room towards the window. All the floorboards are the same width. The ceiling is about 2.5 metres high.



(a) Draw in the missing line between floorboards A and B. (1 mark)

(b) If the floorboards were extended through the back wall for an infinite distance, indicate with an X where they would meet on the horizon. (1 mark)

(c) The vertical dots indicate the sides of a regular door, C, on the right hand wall. Draw an outline of the door, including the top of the door. (1 mark)

(d) A cable runs across the middle of the floor, from left to right. Use construction lines to determine whether the cable runs along P, Q or R. (2 marks)

From Mrs Boughton, Spanish teacher

There are 400 million speakers of Spanish in the world, but the population of Spain is only 46 million.

(a) According to these figures, calculate the proportion of Spanish speakers who do not live in Spain. (2 marks)

(b) Give a reason why your answer to part (a) is probably not the true value for this proportion. (1 mark)

(c) (i) Suggest another language which is spoken outside its country of origin more than in its country of origin. (1 mark)

(ii) Using your general knowledge, give an estimate of how many people in the world speak that language? (1 mark)

From Mr Cuddington, Music teacher

The white notes on a piano keyboard are named using the letters A to G which repeat all along the keyboard. The interval between a particular note such as C and the nearest C above or below it on the keyboard is known as an 'octave'. A regular piano has just over seven octaves. The key indicated in red on the diagram is known as 'Middle C' (even though it's not exactly in the middle of the keyboard). There are four Cs above middle C (high Cs) and three Cs below it making eight Cs in total. These are shown in yellow in the diagram.

The Middle C note vibrates at a frequency of 262 cycles per second (that's 262 *Hertz*.) Every time you go up an octave (from one C up to the next C) the frequency of the note doubles.

(a) What are the frequencies (in Hertz) of the highest and lowest Cs on a piano? (2 marks)

(b) Human ears can hear up to approximately 20,000 Hertz. What would the frequency be of the first C that is too high for humans to hear, and how many octaves is that above middle C? (2 marks)

(c) Going up from a note C to a note G (known as a 'perfect fifth'), the frequency is multiplied by 3/2.

(i) Starting at C and moving up the keyboard by **two** perfect fifths, the frequency of the note is increased by $(3/2)^2$. Show that, after moving up in this way, the frequency of the final note is slightly more than double the frequency of the original note. (1 mark)

(ii) Moving up the keyboard by 'n' perfect fifths, the frequency goes up by $(3/2)^n = 3^n/2^n$. Can this ever be an exact number of octaves? (2 marks)

Answers are given on page 30–31.

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MATHEMATICAL ASSOCIATION

Supporting mathematics in education

Why Isn't There a Maximum Wage?

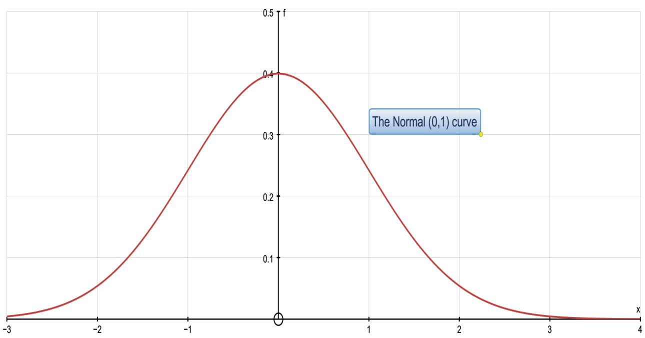
by **Paul Jackson**

The importance of an understanding of statistics is stressed elsewhere in this issue – for example by Dr Tarvin, the History teacher in his Florence Nightingale exam question (page 17), and by archaeologist Sue Greaney who says 'knowing how to interpret statistics is so important' (page 3). I wonder if this skill is stressed enough post GCSE?

In my experience, students usually have an instinctive grasp of the mean, but their understanding of standard deviation can be rather vague. I think the best time to stress the importance of the standard deviation is when the Normal Distribution is studied. If you consider the Normal curve, it is not instinctively obvious to students how far from the mean one standard deviation lies, unfortunately. In fact, it's at the point of inflection, which students who are confident with their calculus can verify by differentiating twice. I suggest this is easier if they start with the probability density function for Normal (0,1) which is

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

and verify that the points of inflection are at $x = 1$ and $x = -1$. I think this is a beautiful result.



If something is Normally distributed, a characteristic such as human height, for example, then the probability of, say, a specific height occurring within one standard deviation (SD) of the mean is about 68%. However, students usually find it easier to grasp that 95% of a distribution lies within 2 SD of the mean, and that 99.7% of a distribution lies within 3 SD of the mean. See table below:

Number of standard deviations from mean	Probability	Approximate frequency of values lying outside this range
1	68.3%	1 in 3
2	**95.4%**	**1 in 20**
3	**99.7%**	**1 in 400**
4	99.994%	1 in 16 000
5	99.999942%	1 in 1 700 000
6	99.9999998%	1 in 500 000 000

Adult Heights

Evidence (and common sense) does suggest that adult heights are Normally distributed (1). The average height of adult males in UK (2) is about 175 cm, with a SD of 7 cm. The equivalent statistics for females are mean 162 cm and SD 6 cm. You can use this to estimate the probability that a UK female selected at random will be taller than a randomly selected male (Answer a). You could also find the limits within which 2, 3 and 4 standard deviations either side of the mean would lie, and discuss what this means (Answer b). There are interesting implications here for the clothing industry, for car manufacturers etc. There is huge scope for some student research on the differences in height between countries and between the sexes in those countries, and whether these differences are reducing over time.

Other Normal distributions

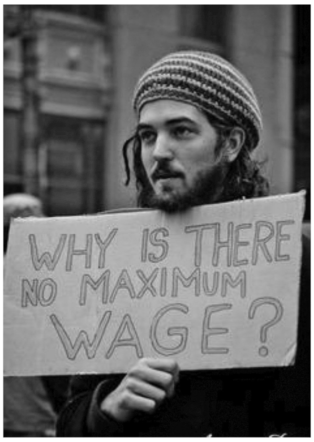
What was said above about height would apply equally well to birth weight, and students could answer similar questions on that. It would apply also to IQ which is one of the more easily measurable aspects of intelligence. Average IQ is defined as 100, with SD of 15. So, again, between what limits do 2 and 3 standard deviations lie? There is no clear definition of a genius (3), but one definition is an IQ over 145. What percentage of the population are geniuses? (see Answer c.)

I wonder if other human aspects, like how hard we are prepared to work also follow a Normal distribution?

This is much harder to define, and harder to measure. Students might like to discuss this, for example they could plot how much time students spend on homework and see if the results are Normal. But there would be such variation in how hard students actually worked in that homework time. This is perhaps a clue in answering the fundamental question posed below:

Why isn't there a Maximum Wage?

I saw the photograph on the next page (4) from a protest march and have often wondered about it. I think it represents something students who study the Normal distribution could consider, and which might help deepen their understanding of standard deviation. The fundamental question to ask is: If many of our natural attributes are normally distributed, shouldn't the rewards we receive be so too? I think this would be a really good question for students to discuss and to ponder on. Let's explore some of the implications. In 2020 the UK minimum wage for adults was set at £8.72 an hour. If we assume a 40-hour working week and a 50-week year, this equates to £17440 per year (5). There will be people earning below this if they don't have a job or are unable to work this much.



The average pay in UK in 2020 is £29660 per year (6). The standard deviation of pay is not easy to find, but students can use their knowledge of standard deviation using the table above to estimate the SD (Answer d). Using your estimate for the SD, where would you set the maximum wage if there were to be one? How many standard deviations above the mean wage do you think is reasonable? The UK population is about 68 million, using the table above, if salary were normally distributed how many standard deviations above the mean should the wealthiest person be? (See Answer e.) Using your estimate of the standard deviation, what would this give as a maximum wage? Do you think it should be this high? Should it be higher? There is so much here for students to discuss, and in the process I think their instinctive understanding of standard deviation can only improve.

Answers

a. 0.079

b. For example only 1 in 400 males would be outside the range 161 to 189cm tall. What is the length of a bed, and what is the probability of an adult being too tall for the bed?

c. 0.135% which is about 1 in 740. There seem to be a lot of geniuses out there. Perhaps it's more about how you use those gifts?

d. You might guess around £6000 to £8000.

e. If wages were normally distributed, 1 in 68 million would be somewhere between 5 and 6 standard deviations above the mean.

References

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2. **https://en.wikipedia.org/wiki/Average_human_height_by_country**
3. **https://sciencetrends.com/what-is-the-genius-iq-level**
4. It is hard to track down where this image is from. If you do an online search for 'Why is there no maximum wage?' this and other images come up.
5. **https://www.gov.uk/government/publications/the-national-minimum-wage-in-2020**
6. **https://uk.jobted.com/salary**
7. **https://www.worldometers.info/world-population/uk-population/**

Keywords: Normal distribution; Standard deviation.

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Piquing Performance with Puzzles

by **Colin Wright**

Decades ago I was at a social gathering when an odd incident occurred. It had emerged that I was studying maths, and one chap turned to me and said: “So, you like puzzles, then?” Incautiously, I said yes, and he went on to say:

> “Here’s a puzzle with a simple solution. You have a wooden sphere and you drill a circular hole through it. The hole is exactly 6 cm from the top rim to the bottom rim. What volume of wood remains?” (1)

Take a moment for reflection: What was your reaction to that ?

Did you immediately start to visualise the object being described?

Did you reach for paper and pen and try to sketch the object?

Did you start to wonder what tools you have that might help you solve the problem?

How would your students react to this puzzle? Perhaps they would react similarly to how they reacted to the now infamous Hannah’s sweets question posed in the 2015 Edexcel GCSE:

> *There are n sweets in a bag. Six of them are orange, the remainder are yellow.*
>
> *Hannah takes a sweet at random from the bag and eats it. Then she takes another sweet from the bag at random and eats that.*
>
> *The probability that Hannah eats two orange sweets is 1/3*
>
> *(a) Show that* $n^2 - n - 90 = 0$ ...

Students took to social media to vent their outrage, many saying that they had prepared thoroughly for the exam and this was nothing like anything they had seen. Some even said it had reduced them to tears.

It’s easy to claim that any capable student should be able to deal with the Hannah’s Sweet question. Start at the beginning, write down things you know, see what you can do with them, simplify, and the answer drops out.

But the sense of outrage is very real, and should not be ignored. These students were caught by surprise, and felt that the question was deeply unfair. Many of these students will have spent hours diligently practising exam questions from past papers, and reviewing the material covered in class.

- How can we help them?
- What’s missing from their otherwise excellent exam preparation?

The very act of having exams as an assessment method will distort the teaching (an example of “Campbell’s Law”). There are then inevitable accusations of “Teaching to the Test”, and claims that students react to trigger words to produce calculations that may or may not be relevant.

However, we know that some students will react with horror at a question that looks unfamiliar, and potentially have their confidence damaged to the point of being irrecoverable. The apparently unresolvable paradox is that students need to be given exam questions they haven’t seen, but they also need to be properly prepared. By “properly prepared”, we don’t just mean to get good grades on their exams, we also mean to prepare them for using their mathematical skills in the real world, where questions and problems don’t come in neat packages that look similar to a hundred examples they’ve seen before, in a context that makes it clear exactly what technique or tool to use.

The challenge at hand is to have something which:

- Promotes resilience;
- Increases engagement;
- Practises some mathematical skills;
- Is feasible in a curriculum pressured setting;
- Is cost effective (time required versus benefits gained).

We know that students can solve equations once they have them, and we know that they can apply a technique when they already know what technique is needed. The help they need is to develop the skills to find the equations, to identify the technique, and to make a start even though the question is unlike anything they’ve seen before. This requires practice and experience, and the resource is not past exam papers, nor a textbook.

Enter the Puzzle... There are many different types of puzzle, and some of them are perfect to help students gain experience, resilience and confidence, and to get them engaged.

Some puzzles require careful visualisation of the exact situation, for example:

> *It takes 12 minutes to saw a log into 3 parts. How much time will it take to saw it into 4 parts? (2)*

Others require a restraint from leaping to do the obvious sums, such as:

> *Three horses are galloping at 27 miles per hour. What is the speed of one horse? (3)*

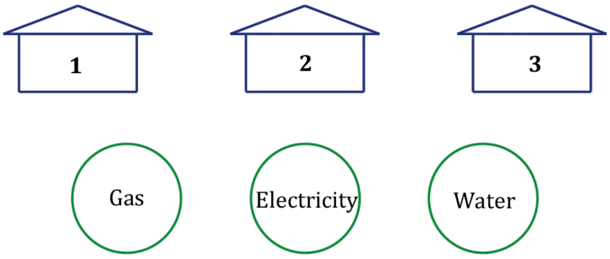
Some puzzles require a careful setup of the formal calculations:

10 kg of cucumbers, that were 99% water, got a bit dehydrated, and became 98% water.

What is their total weight now? (4)

There is even a place for the impossible puzzle exemplified here:

Connect each of three houses to each of three utility supply points, without any supply lines crossing, and without passing under any of the houses. (5)



Why is it impossible? What does 'impossible' really mean? Does this have any useful applications?

And then there are puzzles that are a nod to those students who realise that the real world doesn't always behave like puzzle world.

Ten crows were sitting on a fence.

A farmer shot one. How many were left? (6)

We claim that maths is a genuinely creative subject ... let's allow students to be creative. Of course, an examination isn't the place to do this! But as preparation for exams, as preparation for dealing with problems you've never seen, some creativity is needed, and playing with puzzles is a great way to foster it.

I'm also reminded of the classic case study in which students were posed this nonsensical puzzle:

If a ship is carrying 26 sheep and 10 goats, how old is the ship's captain?

The story goes that students decided that 26 times 10 was too old, 26 minus 10 was too young, 26 divided by 10 was infeasible, so the answer had to be 26 plus 10 ... the captain had to be 36. (More discussion about this problem is available via the link at the end.)

The lesson for students is that when presented with a challenge, they should be prepared to question what it means and whether it makes sense.

The distinction between puzzles and problems is difficult to make precise, but with puzzles there is a sense of "Ooh" or "Ah!", and, when you get the solution, it's not simply the satisfaction of a job well done, there's also some sort of insight to be had. Like a good joke, there's an unexpected twist.

But 'puzzles' can often be the gateway to talking about deeper maths, harder maths, maths that's outside, and even well beyond the school curriculum. They provide an opportunity for students to see that maths isn't just a case of doing calculations and passing exams. It may be that your students already know that, and if so, puzzles may then be a rich source of entertainment, while still exercising their mathematical skills.

Choosing the right puzzles for your students requires some thought, as not every puzzle will appeal to every student. Some will be engaged by a (mildly) humorous setting, while others will want simply to get to the maths. When Dantzig met von Neumann and started to explain the setting of a problem he was working on, Von Neumann snapped "Get to the point!" ... he wasn't interested in the context or setting, he just wanted to see the actual maths.

We need puzzles that are appropriate for the stage of the students. That's hard because each student will have a different balance between their maths skills, reading skills, comprehension skills, and visualisation ability.

And where do we find these puzzles? One possible source would be the UKMT challenges, whose questions are very 'puzzle-like' in their nature. There are excellent puzzle books too, for example by Alex Bellos and Martin Gardner, and teachers often have their own personal collections. Social media is a great way to share them.

The very novelty of puzzles can make them intimidating for many students, and they can certainly be challenging in a number of ways. But that's the point! When we find puzzles that are accessible to students and they can make progress, then confidence and resilience will grow. So let me put in a plea for employing puzzles to promote performance.

Additional information, including links to references and how the story of the sphere unfolded, can be found here:

www.solipsys.co.uk/QL/MA_MiS.html

Answers

(1) This puzzle can be solved using the "Volumes of Revolution" calculation from calculus, or using geometrical methods that date back to Archimedes. But there is a "lateral thinking" approach giving the result without such calculations. This is discussed in the document available above.

(2) 3 cuts are needed to make four parts, so 18 minutes total.

(3) The horse's speed is 27 mph.

(4) The cucumbers now weigh only 5 kg.

(5) It is not possible to solve this puzzle on a plane, or on a sphere. It can be solved on a torus, or on any other surface that has a "hole"

(6) No crows were left because they all flew off after hearing the gun shot.

Keywords: Resilience; Persistence; Confidence; Puzzle.

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A Hundred Years of Past Papers

Reflections on National Examinations in Mathematics

by **Andrew Taylor**

It is often said that everyone is an expert in education because everyone went to school. It is certainly true that views and feelings about mathematics are coloured by our school experience. For many, those few years define what 'proper maths' is. Whilst the emphasis on exams was less important for many students in the past, national examinations have been a barometer of what is regarded as 'proper maths' for many years.

A couple of years ago, I became interested, some might say obsessed, with the history of national examinations in England. Strangely, this started with plans to re-decorate the AQA offices. I was asked to pick three mathematics questions from our archives to decorate a meeting room. As soon as I started looking, it became clear that here was a rich vein of fascinating questions that would be interesting to teachers and anyone involved in mathematics education. So I started tweeting images of some questions starting with this one from 1917.

> 7. A British soldier marching in quick time takes 120 paces of length 30 inches to the minute, and at double time 180 paces of length 40 inches to the minute. Two men start at the same time to travel 6 miles, A at military quick time, B at 4 miles an hour. At what distance from the end must A quicken to double time so that, retaining the latter pace to the end, he may just overtake B?

From looking at old questions, I moved on to looking at content, structures and results for historic qualifications which led to a workshop at a LaSalle Complete Mathematics conference in March 2019. This article is based on the content of that workshop.

History

The first national examinations in England were the School Certificate and Higher School Certificate introduced in 1917. In 1951, these were replaced with the General Certificate of Education at Ordinary and Advanced level (O and A-levels). O-levels were aimed at around the top 20% of learners and, in 1965, a new examination was introduced to offer nationally-recognised qualifications to a wider range of 16-year olds. This was the CSE (Certificate of Secondary Education). In 1988, CSE and O-level were replaced by a single examination at age 16, the GCSE. More recently we have seen substantial reforms of both GCSE and A-level leading to the current examinations.

What's the same and what's changed?

(1) Content

I often hear, and sometimes use, the phrase 'mathematics is mathematics' implying that the stuff of secondary mathematics is essentially constant. It is certainly true that much of what we teach now would be recognisable to a student at school in the middle of the last century but the emphasis on what we value and how we assess mathematics has shifted markedly.

The Joint Matriculation Board (JMB) O-level syllabus of 1958 was only about six pages long and leaves a great deal of room for interpretation of the content. For the reformed 9-1 GCSE, AQA's specification of content is 28 pages long and is supported by a Teaching Guidance document that runs to over 200 pages. Contrast this with 1958 where number and arithmetic is dealt with as follows

> *The ordinary processes of Arithmetic. The commoner systems of weights, measures and money, including metric units. Fractions, decimals, ratio, proportion, percentage. Use of common logarithm and square-root tables. Significant figures.*

This is followed by equally concise information about mensuration and trigonometry and a brief paragraph on algebra, starting with the wonderful phrase 'Algebra to quadratic equations'. The relative importance of different content areas is then made very clear with a full four pages given over to Geometry, offering great detail on required proofs and constructions.

In terms of how content was tested, I guess it is no surprise that arithmetic questions from 50 or more years ago look very different. Students had to deal with the complexities of Imperial measures without the aid of calculators in questions like this from 1957:

2 (i) If certain goods cost £9 6*s*, 8*d*. per ton what is the price of 19 cwt. 3 qr. 14 lb.?

(ii) What is the length of the side of a square which has the same area as a rectangle with sides 2 ft. 4 in. and 28 ft. 7 in.?

In contrast, many algebra questions are very familiar and could be asked now with only slight changes. This question from 1978 could have been set, without looking

out of place, any time in the last 60 or 70 years:

A1 (b) Solve the simultaneous equations

$2x - y = 5$
$7x + 2y = 1$

One striking feature of older papers is the absence of diagrams. Students were expected to deal with lots of information presented in lengthy sentences and choose for themselves whether a sketch would be helpful. These two trigonometry questions from the 1950s would almost certainly be supported by diagrams if asked in a GCSE paper today. The first question is one that I could imagine appearing in the current GCSE. The second is a style of question used regularly in GCSE Higher papers up to ten or so years ago when angles of elevation were specifically included in GCSE specifications:

6. A rectangle has sides 10 in. and 17 in. long. Calculate the acute angle between its diagonals.

11. From a point P on the same level as the base of a tower, the angle of elevation of the top of the tower is 32° 14'. At a point Q, 15 ft. vertically above P, the angle of depression of the base is 5° 12'. Find (i) the height of the tower, (ii) the angle subtended by PQ at the top of the tower.

(2) Structure and optionality

Although the specification of content was brief, examination time wasn't. In 1958, the O-level examination was 6 ½ hours long with separate papers on Arithmetic, Algebra and Geometry. Even this is a sprint compared to the London Board exam of the time. Here, no fewer than seven papers were offered, each 2½ hours long. Three of them were compulsory and a fourth was optional, so ten hours of exam time in total. The optional paper on the History of Mathematics is, without doubt, the least familiar of all the old papers I've come across. I'm not sure what today's sixteen year olds would make of this from 1957:

2. Why can Archimedes be regarded as the originator of the integral calculus? Who could be regarded as the originator of the differential calculus?

Optional papers are, of course, familiar to us in A-level though not GCSE. Throughout the life of O-levels, wide choice in syllabuses, papers and questions was very much the norm. In 1978, for example, JMB data shows no fewer than 13 mathematics and additional mathematics syllabuses being offered including specific examinations for curriculum development groups such as MEI, SMP, MME and the Schools Council, and a 16+ exam. The last of these syllabuses was a collaboration between JMB and CSE boards to produce a single examination intended for a wide range of learners; essentially a forerunner of GCSE ten years in advance. This makes an interesting contrast with the speed of introduction of qualification reform in recent years.

Within most O-level papers, there were sections of compulsory, usually relatively short, and optional, longer questions. For example, JMB's syllabus C in 1978 was made up of two papers, each 2½ hours long. Each paper had a section A of 20 compulsory questions totalling 55 marks and a section B where each question was worth 15 marks and three had to be selected from five available.

All this optionality does throw into question the comparability of standards 40 years ago. A student sitting O-level, or at least their teacher, had a choice of exam boards that was much greater than today; a choice of syllabuses with different structures, exam duration and content; and a choice of which topics to focus on when preparing students for extended questions. It is certainly true that some O-level exams were considered easier than others and such a view probably had greater truth than it does in our, rightly, highly-regulated exams system.

It would be interesting to know if examination structures have affected long term attitudes to maths. The emergence of calculators in class and in exams is, perhaps, the most obvious 'watershed' but are there subtler changes that have had their effect? Many people in their late 20's and 30's will have been assessed by modular exams whilst those of us educated in the 1970's and 1980's were subject to the O-level/CSE divide. Has the much-quoted motivational effect of modularity carried through to adulthood? Has the stigma of not being considered up to O-level lasted through some people's working lives?

In my trawl though the archives, I've found many great questions and some awful ones. I've seen interesting contexts, contrived contexts and many archaic contexts. I've come across questions that could sit comfortably in a GCSE nowadays and others that are very much of their time. Like anything, the content of mathematics exams is subject to fashion and I do miss some of the content that is currently out of fashion. However, a great question remains a great question and I will finish with this one from a 1940 School Certificate paper which is a favourite of mine.

The area of a right-angled triangle is 120cm^2.

The hypotenuse is 8cm **less** than the total length of the other two sides.

Find the lengths of the three sides of the triangle.

Keywords: Examinations; Past Papers.

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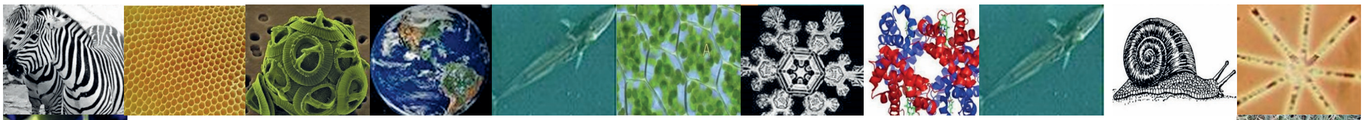
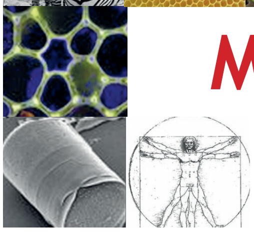


Mathematics is Everywhere

by **David Hunt**

Introduction

When David Layton (1978) described the following as the hidden curriculum in mathematics, I had hoped that by 2021 the situation may have improved.

- Firstly, mathematics is wholly good; it is a worthwhile activity for its own sake, irrespective of utility, there is an intrinsic merit in the ability to show, for example, that the square of every even number can be expressed as the sum of two consecutive odd numbers.
- Secondly, mathematics is largely done in silence and solitude.
- Thirdly, mathematics is done with pencil and paper in schools.
- Finally and perhaps the biggest indictment of all, mathematics is inescapably boring and unremittingly difficult.

"And so say all of us", I can hear being chanted in mathematics classrooms up and down the country. Generations have been failed by a product approach to the teaching of mathematics. In many cases this continues.

> "Pupils' learning is based too much on their acquisition of methods, rules and facts, and too little on their understanding of the underpinning concepts, on connections with their earlier learning and other topics in mathematics, and on helping them make sense of mathematics so that they can use it independently."
>
> (Ofsted, 2008)

> "It remains a concern that secondary pupils seemed so readily to accept the view that learning mathematics is important but dull. They frequently told inspectors that in other subjects they enjoyed regular collaboration on tasks in pairs or groups and discussion of their ideas, yet they often did not do so in their mathematics lessons, or even expect to do so."
>
> (Ofsted, 2012)

Little has changed with the introduction of the latest National Curriculum in 2014.

> "Teaching overall remains variable within and between schools:
>
> - The best develops reasoning (through discussion, questioning, careful choice of tasks, exercises and problems), and provides challenge.
> - The weaker continues to teach methods like recipes with lack of attention to reasoning and problem solving."
>
> (Ofsted, 2017)

Improving the delivery of the mathematics curriculum

This reminded me of a conversation that a good friend of mine had had with his Dad (a qualified precision engineer and a Member of the Institute of Production Engineers). The conversation in the 1960s something like this:

> *We were sat at the dinner table and he pointed to the drinking mug I had in front of me and asked what it was. Greatly confused that my own father didn't know that it was a mug; I looked at him with an amount of confusion written all over my face. I didn't answer but he asked again "What is that?" pointing at the mug. I obviously said "It's a mug". He simply said, "No it isn't" and left the table. Bear in mind of course that I was just a young boy at this point and I had no idea what he was talking about. He then put something else in front of me (I think it was a book) and asked the same question. I said "A book", he said "No it isn't". This happened, I think, another couple of times and I then started thinking this was simply a trick question. I didn't know what the trick was or what the solution was.*
>
> *It was a few minutes later that he put a glass in front of me and asked "What's that?" By then I had worked out the first part and said to him "I don't know but it isn't a glass is it?" He said to me that, "As well as all the objects I have shown you and every other object I haven't shown you, this is not a glass it is mathematics". He then sat down with me and explained that everything I see, everything I do, everything in Leeds, Yorkshire, England, in fact the universe is in fact mathematics. He then went on to prove it.*
>
> *How many glasses are in front of you – mathematics*
>
> *How much liquid does it hold – mathematics*
>
> *Where do we live – how long have we been here – how high is the wall and how far is it to the sun – how much air do we breathe – how many toes have I got, ...*
>
> *I got the message. I couldn't help but get the message because he was right and it was probably one of the only statements we will ever hear that is likely to be the absolute truth.*

(My friend's dad was being pedantic by only accepted the answer mathematics rather than both, e.g. glass and mathematics.)

Step forward 60 years and the International Day of Mathematics (IDM) for 2020 had the theme "Mathematics is Everywhere". The IDM website has more information, including downloadable activities; you'll find it at **www.idm314.org/math-everywhere-video.html**.

Examples include:

Mathematics is in ... chicken eggs!

Submitted by Bağcılar Enderun, Gifted Children's Center in Istanbul, Turkey.

Mathematics is in ... a cup of coffee!

Submitted by Pedro Morales Almazan, from Santa Cruz, California, USA.

Mathematics is in ... jumping rope!

Submitted by the Colégio Verde Água *from Mafra, Portugal.*

Problem solving and reasoning skills need to be at the heart of the mathematics curriculum. Teachers of mathematics and their students need to understand that "mathematics is a problem solving process that is of benefit to them in their own lives and their chosen career or future study" (Hunt 2018). Problems need to be set where students can use and apply their mathematical knowledge or develop further mathematical knowledge in order to solve the problem.

It is not the topic that is important but the realization by students that mathematics is indeed everywhere, and as such is important for the student to understand that, and how mathematics is used to solve real life problems. The mathematics content can then become a "toolbox" in the mind of the student, out of which the student brings the necessary mathematics to solve the problem (Ofsted, 2012). For example, the final solution to solving a problem might involve solving an equation, or drawing a graph.

Mathematics is everywhere examples

If mathematics is indeed everywhere, then we as teachers of mathematics should develop in our students the ability to think where the mathematics is in a certain situation. Students need to be given the opportunity to develop these process skills. This can then be linked to applying mathematics across the curriculum, so that all teachers understand that mathematics is everywhere and can be developed within their curriculum area.

An example might be for students to investigate the mathematics involved in a topic they are interested in. It could be a sport or a television programme, Objects or photographs may be used for students to investigate the mathematics behind an object or photograph.

The photographs at the start of the article are taken from NRICH (nrich.maths.org) under the title: Mathematics is everywhere if you look carefully enough.

> *Look at these 16 images. How many can you identify? What mathematics can you see in the images? Can you think of a reason for the shapes existing as they do? Can you spot shapes which share any common mathematical structures?*

Further examples can be found at MathsCareers.org.uk. They include how trigonometry helps to solve murders and how mathematics saves lives by helping to fight cancer.

The present situation with the coronavirus pandemic has made it necessary for schools to implement processes and procedures to keep staff and students safe. It would be a useful discussion for students to realize the mathematics that has been involved in the production of such processes and procedures. There is also a wealth of statistics involved with the pandemic and this could be discussed through the worldwide pandemic down to the effect of the pandemic in the school they attend and the area where the students live.

School Inspection Handbook (Ofsted 2019)

The 2019 Ofsted education inspection framework (EIF) paragraph 300, applying the EIF to the teaching of mathematics says this:

- the school's curriculum identifies opportunities when mathematical reasoning and solving problems will allow pupils to make useful connections between identified mathematical ideas or to anticipate practical problems they are likely to encounter in adult life.
- pupils' mathematical knowledge is developed and used, where appropriate, across the curriculum. Ofsted, 2019)

This should give further impetus to use the examples given in this article.

Conclusion

I end with what I hope David Layton would be able to write about the mathematics curriculum in the 2020s.

- Teachers have high expectations of pupils' enjoyment and achievement
- Teachers make a conscious effort to foster a spirit of enquiry, developing pupils' reasoning skills through approaches that see problem-solving and investigation as integral to learning mathematics
- Teachers checked that everyone is challenged to think hard and they adapt how they were teaching to achieve this
- As a result, their classrooms are vibrant places of learning.

(Ofsted 2008)

"And so say all of us", I want to hear being chanted in mathematics classrooms up and down the country.

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Keywords: Curriculum; Reasoning; Problem solving.

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Becoming a Mathematical Ninja

by **Colin Beveridge**

It was the first class of Year 12. Mr Rowley, writing on the blackboard, said

"$\frac{7}{\sqrt{3}}$, which is…", and the entire Further Maths class reached for a calculator. (This was in the olden days, like, 1994 or so.)

The very quickest of us had pressed the 'on' button before he tutted and said "4 and a bit. 4.04."

I don't think he'd set out to impress us – as the course went on, it became clear that estimating numerical answers was pretty much a compulsive habit for him – but we were eating out of his hand from that moment on. We reasoned, if someone was *that* good with numbers, they must be *that* good at maths. And that's the premise of this article: if you can learn a few number tricks that *look* impressive but turn out to be straightforward, you can take a huge shortcut on the road to convincing your students you know what you're doing. Obviously, you don't *need* to learn any of these tricks, but I promise: the look of absolute terror and disbelief when you reel off an accurate answer in your head is well worth the effort.

So, how did Mr Rowley do it? I don't know for sure, but I have some good guesses. I surmise that he had two relevant facts at his disposal:

- The square root of 3 is about $\frac{7}{4}$; and
- That estimate is about 1% too high.

Seven divided by 7/4 is four. If we reduce the denominator by 1%, that's roughly the same thing as increasing the numerator by 1% - and increasing the answer by 1% gives 4.04.

The maths here isn't at all difficult! The only tricky things are knowing the numbers and having the confidence to trot them out quickly. But who has time to memorize great big lists of square roots? Certainly not busy teachers. Fortunately, estimating square roots is also fairly quick and simple.

Estimating square roots

Suppose you had to estimate $\sqrt{18}$, and for some reason $3\sqrt{2}$ wasn't an acceptable answer. (Of course, if you know that $\sqrt{2}$ is on the high side of 1.41, you can say $3\sqrt{2} \approx 4.24$ immediately. But let's pretend you don't.) I have two words for you. Word # 1: Newton. Word # 2: Raphson.

Wait! Come back! You don't have to do the whole differentiation thing in your head every time – in fact, once we've gone through the theory here, you can just learn the trick and forget about the evidence.

I'll recap the method quickly: if x_0 is a good estimate for a solution to $f(x) = 0$, then

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$ is usually a better estimate.

The function we're looking at here is $f(x) = x^2 - 18$, which has a zero when $x = \sqrt{18}$. The derivative is $f'(x) = 2x$.

And we can readily ballpark the number: the square root of 18 is between 4 and 5, and towards the lower end of that interval. Let's use $x_0 = 4$ Then

$$x_1 = 4 - \frac{16-18}{2\times 4} = 4.25.$$

That's not too shabby!

When you look at the calculation, you might notice that the top of the fraction is the difference between the square you've picked and the target number; the bottom is just double your estimate. So, if we anticipate that we're subtracting a negative, the simplified Newton-Raphson recipe for square roots is:

- Pick the nearest square number whose square root you know (in this example, that's **16** = 4^2.)
- Work out the signed difference between your target and this square (18 is **2** more than 16).
- Divide this by double the perfect square root (2/8 = 1/4).
- Add this on to the initial guess (the final answer is 4 + 1/4 = 4.25).

With a little practice, this becomes routine. The square root of 34? It's a bit short of 6. Two-twelfths short, so it's about five and five-sixths, or 5.83. For the more algebraically-minded:

$$\sqrt{a^2+b^2} \approx a + \frac{b}{2a},$$

when b is much smaller than a, because

$$\left(a+\frac{b}{2a}\right)^2 = a^2 + b + \frac{b^2}{4a^2}.$$

The last term is, by assumption, comparatively small.

Fractions

I believe it's important to bring your influence to bear on one's students. I was especially proud when I got a message from a former student saying "The answer to one of today's problems was 5/13. I rattled off 0.384 615 and the whole class said 'gosh'."

So how did they do it? This trick rests on a lovely number fact and a dirty trick. The number fact is that

$7 \times 11 \times 13 = 1001$, which means that $\frac{n}{13} = \frac{77n}{1001}$.

And that's relatively easy to get a good estimate for! If $n = 5$, then $77n$ is $35 \times 11 = 385$, and 0.385 is a solid estimate for 5/13.

But we can do better! We wanted to divide by 1001, not 1000, so our answer is about one part in a thousandth too large. If we take away 0.000385, we'll be even closer.

Unfortunately, subtraction is a bit fiddly – unless you are in possession of a dirty trick called the nines' complement.

The nines' complement

Think about how you would work out 1 – 0.385. Depending on where you learnt to subtract, you would probably write out 1.000 above 0.385, then "borrow" repeatedly until you had several nines on top and a final 10 on the right; you could then work out 0.615.

The thing is, if you know you're subtracting from 1, there's little point in going through the motions of writing the whole thing out. You're subtracting each digit from 9, except the last, which you subtract from 10. Once you notice that, you can reel off the answer at a glance: 0.615. (Strictly speaking, this is one more than the nines' complement of 0.385 – to get the nines' complement, you simply subtract each digit from 9.)

And if you want to take 0.000385 from 0.385, all you need to do is drop the final digit of the larger number by 1 and stick the nines' complement on the end to get 0.384615. (In fact, 5/13 is precisely these six digits recurring.)

Sevenths and elevenths

Slightly less impressively, and with slightly more overkill, you can use almost the same trick for sevenths:

$\frac{n}{7} = \frac{143n}{1001}$.

For example, $\frac{4}{7} = \frac{572}{1001}$, which – in an instant – is 0.571428. Again, the fraction is precisely these six digits recurring. (You could, of course, just remember the rotating string of digits for sevenths. But where's the fun in that?)

A similar trick also works for elevenths, but this is *definitely* overkill!

Trigonometric values

I can specify the exact moment I got hooked on mathematical ninjary. Standing in at a homework help centre, I noticed that the week's homework sheet was full of 3-4-5 triangles. The same angles kept showing up on the students' calculators. After a few dry runs in my head, I casually explained "… which is about 53.13 degrees…" and suddenly the table was convinced I was a genius. All I'd done was remember an angle!

I have three broad classes of tricks for trigonometry: depending on where the angle you're interested in lies, I use either a small-angle trick (for angles below 30°), a mid-range-angle trick (30° to 60°), or a transformation (60° and above). (I'll only be looking at angles between 0° and 90° here. It's not that hard to extend to different angles using symmetry.)

Small-angle tricks

In *radians*, small-angle approximations are very straightforward:

- $\sin(x) \approx x$
- $\tan(x) \approx x$
- $\cos(x) \approx 1 - \frac{1}{2}x^2$

(OK, the last one is less straightforward.)

Unfortunately, the majority of trigonometry encountered day-to-day is done in degrees, so if we want to impress, we probably need to convert. Fortunately, that's not too hard to do, in a rough-and-ready sort of way: one radian is about 60°, so for the sine and tangent of small angles, dividing the number of degrees by 60 gives a fair approximation.

Of course, a radian *isn't* quite 60°; that's an overestimate of about 5%, so we can improve our sine and tangent estimates by adding on 5% if we feel it's worth it. For example, $\sin(15°) \approx 0.25$, because that's 15/60; if we add on 5% of the answer, we get something in the region of 0.26, which is better. (The smaller the angle, the better; the tangent approximation is generally worse than the sine one.)

Cosine needs a few more steps, but benefits from a numerical coincidence:

$\frac{\pi^2}{180^2} \approx 0.0003$.

That means that converting degrees-squared into radians-squared is as simple as multiplying by 3 and moving the decimal point four spaces to the left.

That makes the recipe for the cosine of small angles:

- Square your angle
- Add 50% to that answer (which accounts for the half

in the small-angle approximation, as well as the 3 in the conversion)

- Subtract the result from 10,000
- Divide by 10,000.

For example, to work out cos(12°), you would square the angle (144), add 50% (216), subtract the answer from 10,000 – the nines' complement works nicely here to give 9784; your estimate is 0.9784, which is good to three significant figures.

All of these processes can be reversed, too; to find $\sin^{-1}(0.1)$, you might multiply the argument by 60 and then drop the result by 5% to get 5.7°, which is correct to two significant figures.

Large angles

The best trick I have for large angles is to turn them into small angles by making use of symmetry. Complementary angles fit together,

$\sin(x°) = \cos(90 - x)°$

and vice versa, so rather than work out cos(83°), you'd do better to work out sin(7°), which gives the same value, around 0.12. Tan is slightly harder:

$$\tan\left(90-x\right)° = \frac{1}{\tan\left(x°\right)},$$

so you will need to be a bit sharp on your reciprocals here; alternatively, you can divide 60 by your small angle and subtract 5% from the result: to work out tan(84°) do and 60/6 = 10 and reduce the answer by 5% to get 9.5, which is correct to 2sf.

Mid-range angles

Except for the special values, the mid-range values are typically quite hard to calculate accurately; however, if you know a few waypoints and can interpolate reasonably well, you can come up with passable estimates.

I find the 3-4-5 triangle, which has angles of 36.87° and 53.13°, to be especially useful, as these angles are close to midway between the special angles of 30°, 45° and 60°.

For example, if you wanted to know sin(50°), you might say "sin(45°) ≈ 0.71 and sin(53°) ≈ 0.8, so 0.76 looks like a good bet. (It's closer to 0.77, but that's definitely in the 'good enough' range.)

It's worth noting quickly that the cosine estimate formula from the previous section still works pretty well for mid-range values, combining tricks from the last two sections, you might instead work out that sin(50°) = cos(40°) Squaring 40 gives 1600; adding 50% takes you to 2400, the nines' complement of which is 7600 – giving 0.76 as the final answer again.

Summary

My aims in this article were to convince you that quick mental maths tricks can have an extremely high impressiveness-to-effort ratio, and to set you on the road towards learning a few. Once you have a few under your belt, your working memory will improve and open the door to even more eyebrow-raising feats of arithmetic. You may even develop your own tricks – in which case, I want to hear about them!

Keywords: approximation, mental methods

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ANSWERS TO GUEST EXAM QUESTIONS

by **Rob Eastaway**

(Remember this is a thought experiment, NOT a formal exam mark scheme!)

HISTORY

(a) January 1855

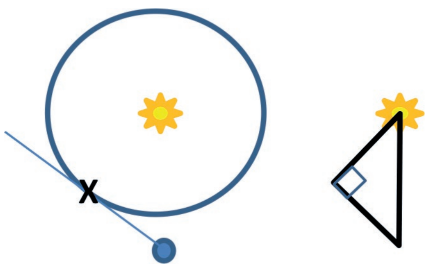
(b) Far more soldiers were dying from disease than from wounds in battle, so prevention of disease was a priority. Deaths were seasonal, there were far more deaths in winter than summer, so more nurses/ resources were needed in winter.

(c) More fighting leads to more wounds, and the months with the most wounds were October, November, December. This assumes that people die from wounds within, say, a week of a battle. In Nov 1854, there are two orange segments. The orange segment is larger in this month than any other, which suggests more people died from wounds that month than any other month, perhaps a similar number to the number who died from disease. However, there is no green segment that month. Did Nightingale forget to colour the middle segment green?

(d) There is no scale. Are we supposed to measure deaths by the area of a segment or by how long the side of a wedge is? In November 1854 the area of grey is larger than the area of orange (more disease), but the orange wedge has a longer side than the grey segment (more wounds). The labelling does not make it clear which interpretation is correct.

PHYSICS

(a) Tangent



(b) The tangent to a circle is at right angles to the radius, so the angle between Earth-Venus and Venus-Sun is 90 degrees at this point.

(c) The radius of Venus's orbit is r. When the angle between Venus and the Sun is 45°, Earth/Venus/Sun form an isosceles right-angled triangle, and using Pythagoras, the distance from Earth to the Sun is therefore $\sqrt{2} \times r$. (This is close to reality, by the way.)

PSYCHOLOGY

(a) Starting with the secret number N:

Double it	2N
Add 10	2N + 10
Divide by 2	N + 5
Subtract N	5

(b) Children are surprised because they are not used to getting the same answer whatever number you start with. Different numbers usually lead to different answers. They also know that their number is secret, and having the teacher tell them the number they finished on makes them feel like their mind has been read.

(c) Surprises are things that are unexpected. A maths surprise might be:

- ✓ Something that happened in a lesson. Perhaps the teacher announced you were going to do an activity that you weren't expecting e.g. they brought in a puppet, or decided to show a cartoon; or maybe it was a bad surprise, such as a test.
- ✓ Something in the maths itself, for example after doing a routine, repetitive task, a pretty pattern emerged.

Surprises often arouse curiosity. They make you want to know WHY something happened, and hence they can motivate you to want to learn more. However not all students enjoy surprises in maths. If you thought you understood something, and then discover that you were wrong, a surprise can undermine your confidence.

(NOTE: If this question were really asked in a maths exam, most students might struggle to recall any credible examples of maths surprises. This isn't necessarily the students' fault. It might be that their experience of maths has been almost entirely devoid of surprises.)

PE

(a) Maya runs two straights of length L, and a circle of radius R, so her entire lap length is 2L + 2πR.

(b) Kelly runs in the second lane, so the radius of her circle is R+1 metres. Her total lap is therefore:

$$2L + 2\pi(R + 1) = 2L + 2\pi R + 2\pi.$$

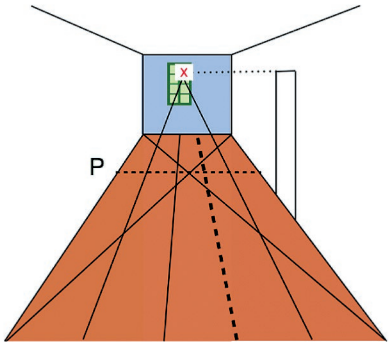
Subtract Maya's distance

$$2L + 2\pi R + 2\pi - (2L + 2\pi R)$$

everything cancels to leave an extra distance of 2π, so Kelly runs about 6 metres more than Maya (or 6.28 metres if you want to be more precise).

This problem is based on a true story: in the Athens Olympics in 2004, Kelly Holmes ran the whole final lap in the second lane, and still managed to win gold medal.

ART & DESIGN



(a) The thick dotted line shows the line separating floorboards A and B.

(b) All floorboards and the lines along the ceiling meet at the vanishing point X.

(c) The top of the door also lines up with the vanishing point X. A regular door is about 2 metres tall, so roughly 80% of the height of the wall.

(d) Join the corners of the floor with straight lines. The two lines cross halfway along the room. The cable therefore runs along the line of P.

SPANISH

(a) The proportion of Spanish speakers outside Spain is 354/400, which is a little less than 90%.

(b) The numbers 400 million and 46 million have clearly been rounded, so the real figure is higher or lower than that. Also, these numbers depend on what is meant by a 'Spanish speaker'. Is it somebody who can get by with a few Spanish phrases, or somebody who is fluent? And just because you live in Spain doesn't mean you speak Spanish – lots of Brits who live on the Costa del Sol can vouch for that. So the number of Spanish speakers in Spain is probably lower than 46 million.

(c) (i) One example is English. There are roughly 70 million in UK, but many more English speakers in continental Europe, Asia, America, Australasia and Africa. Another example is Portuguese (spoken in Brazil).

(ii) It is thought that between 300 million and 400 million people in the world speak English fluently and up to four times that number to some extent.

MUSIC

(a) Top C is 4192 Hertz. Bottom C is 32.8 Hz.

(b) The first inaudible C would be 33,536 Hz, which is seven octaves above middle C.

(c) (i) $3/2 \times 3/2 = 9/4$ which is 2.25.

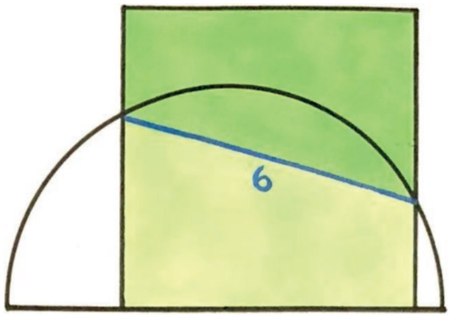
(ii) 3^n is an odd number and will never be exactly divisible by 2^n, so however many perfect fifths you move up by, the frequency will never have increased by an exact power of 2, and hence cannot be a whole number of octaves. This interesting but challenging number property is one for your top A* students to appreciate.

Geometry Problem 8

by **Catriona Shearer**

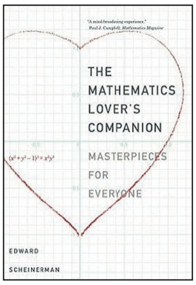
A chord, of length 6, splits the square in half.

What's the area of the semicircle?



Send your solutions to Chris Pritchard (**chrispritchard2@aol.com**) and we will publish the best.

The Mathematics Lover's Companion
Edward Scheinerman
Yale University Press
www.yalebooks.edu
ISBN: 978-0-300-25539-3
296 pages.
Price £12.99 (paperback)



One of the joys of reading 'popular' mathematics titles is that ideas which may already be familiar in terms of their mathematical content are often presented from a fresh perspective. In this agreeable volume, Scheinerman takes the reader on a tour of many of the topics which have inspired so many mathematicians and learners alike over the years. The style is conversational but authoritative and is accessible to students in the later years of schooling or anyone wishing to revisit fascinating mathematical topics through the lens of another mathematician's experience.

The author points out that collections of mathematical topics in a book is not something new and that anyone choosing such a set of topics will end up with a list of their personal favourites, but the stance taken here is to ensure that each topic fits a number of criteria. Firstly, the mathematics may not be known to non-mathematicians, or at least not in any depth, perhaps only an awareness of a topic, such as prime numbers or fractals. This text offers the reader some insights into each topic that they may not have considered. There is a thread throughout of proving results, often using reductio ad absurdum, otherwise known as proof by contradiction. Each topic covered brings an element of surprise and discovery, and there is a real-life context for each topic. The book can be read from cover to cover, but some chapters or groups of chapters can be read independently.

The opening section has several chapters focussing on properties of numbers, starting with prime numbers and moving on to consider representations in different bases and some of the anomalies that can arise. Each of the special numbers √2, π, e, and ∞, the symbol for the infinite is given its own chapter, as well as consideration of factorials, Fibonacci numbers and the ever-intriguing Benford's Law. The chapter on √2 takes a look at how the number is important in music scales which gives rise to harmony and discord and the musical keys we now use, and the chapter on π explores some of the interesting ways the number was derived by different mathematicians.

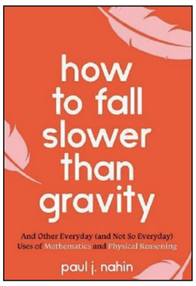
The middle set of chapters focuses on shape, starting with results about triangles, including Pythagoras and the link to Fermat's Last Theorem and covers the different centres of triangles in a succinct way. Then follow chapters covering circles (including circle packing) and the Platonic Solids before stretching the reader a little with chapters on fractals and hyperbolic geometry.

In the final section of the book, Scheinerman considers some topics less likely to appear on the curriculum, but which nevertheless offer rich content to raise curiosity and intrigue. In the realm of probability, a chapter on non-transitivity tests the intuition followed by applications of probability in a medical context. This chapter in particular has relevance to the current world context and how testing for Covid or other conditions can be less than the accurate science we might expect. The book ends with chapters on chaos, Arrow's Theorem and its application to election systems and finally Newcomb's paradox which often forms the basis of many television game shows.

Already having a bookcase of mathematical books covering many of the topics in this volume, I was expecting 'more of the same', and yet I was pleasantly surprised to read this volume and enjoyed Scheinerman's easy style but he uses sufficient rigour to maintain a degree of challenge in a very readable source of mathematical delights!

Ray Huntley

How to Fall Slower than Gravity
Paul J. Nahin
Princeton University Press
6 Oxford Street, Woodstock,
Oxfordshire. OX20 1TR
https://press.princeton.edu/
ISBN: 978-0-691-22917-1
320 pages.
Price £14.99 (paperback)



This book is composed of 26 problems, each one intriguing and perhaps 'odd' in the sense that it is not a textbook situation. The book starts with a preface – yes, most of us ignore this completely, but this one is worth reading and working on the challenges set. The author argues against a comment in a paper which asks the use of most mathematics. Similar in a way in which pupils or students of non-mathematical subjects might query their value. His argument leads to the collection of essays in the book as to how and where mathematics is without doubt of use and his focus is on algebra, trigonometry, geometry and calculus in a physical application.

The distinction between mathematical physics is not sharp and therefore these applications are posed in physical contexts using mathematics mainly at school level, but you will need to think and that is what puts our subject into a different arena.

What's it all about? Very simply, 26 problems of varying standards, mathematics and context. Each has a solution at the back of the book like all good mathematics books! Some of the mathematics is straightforward although the ideas behind the process can be very testing while some is non-trivial and requires some 'good' manipulations and heavy thinking. Exactly what is expected in a decent mathematics text.

The solutions deserve a mention. These are fully explained with many over 2/3 pages and equations/ideas with full explanations annotated at various sections. Thus they are actually readable, but it is much better to attempt first.

What makes these problems interesting is twofold. First of all, their direct application of some part of real life – not an invented application which pupils see through – but an actual problem. Secondly, each problem connects more than one area of mathematics. For example, the proof of the Cauchy-Schwarz inequality in the Preface (as this is required in later problems) links integration, algebraic expansion of functions and some quadratic theory, all very straightforward in their own way, but when connected they do require a bit of thought.

There are four teasing appendices: two on continued fractions (one on the use of MATLAB and one which is a school level explanation of Brouncker's continued fraction for $\frac{4}{\pi}$. The third is a calculus solution to the depressed cubic equation and the last Lord Raleigh's solution to a rotating ring problem. The book closes with a comprehensive seven page index.

This book is suitable for senior pupils at A-Level especially those studying Further Mathematics. It is also suitable for IB pupils and should give plenty of ideas for their Exploration topic. It is a good investment for every department for those who need stretched or are curious about mathematics outside a classroom situation.

N. G. Macleod

MATHEMATICS IN SCHOOL

NOVEMBER 2021
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MATHEMATICAL ASSOCIATION
150th ANNIVERSARY 1871–2021
Supporting mathematics in education

Authors' notes

Mathematics in School is aimed mainly at teachers of school and college pupils of 10 to 18 years of age and for those working with students who are preparing to enter the teaching profession.

We attempt to attain a balance of articles reflecting this age and ability range and look for pragmatic articles; ready-to-use materials; discursive, possibly philosophical articles; speculative, reflective, and sometimes retrospective pieces. Newsworthy items have a place, but the average lead time sometimes precludes this. However, book and equipment reviews have a very important role to play. There is also the opportunity to stimulate – and even amuse – otherwise hard-pressed and very busy teachers. If you can submit an original article (i.e. usually, one that has not been published elsewhere) that fits this brief, then the Editors will be very happy to consider it for publication. Few articles used are over 4 pages long when typeset. Half-page 'fillers' are always welcome. They can be odd hints and tips, items of news, letters, cartoons or photographs with one-line captions. Please note that this is not a refereed journal, but a second opinion is usually sought. An article may be rejected, but a suggestion may be made as to where else it could be submitted. An Editor may suggest changes, but that in itself cannot guarantee eventual publication. Naturally an Editor may also cut or otherwise modify an article themselves, but the Author always has the opportunity to see the result at the proof-reading stage. Prompt return of the corrected proof is essential.

Always include at the top of the article:

Title

Author(s) names in full.

Spelling *Mathematics in School* uses British spelling, according to *The Oxford Dictionary of English*.

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Acheson, D. 2020 *The Wonder Book of Geometry*, Oxford University Press, Oxford.

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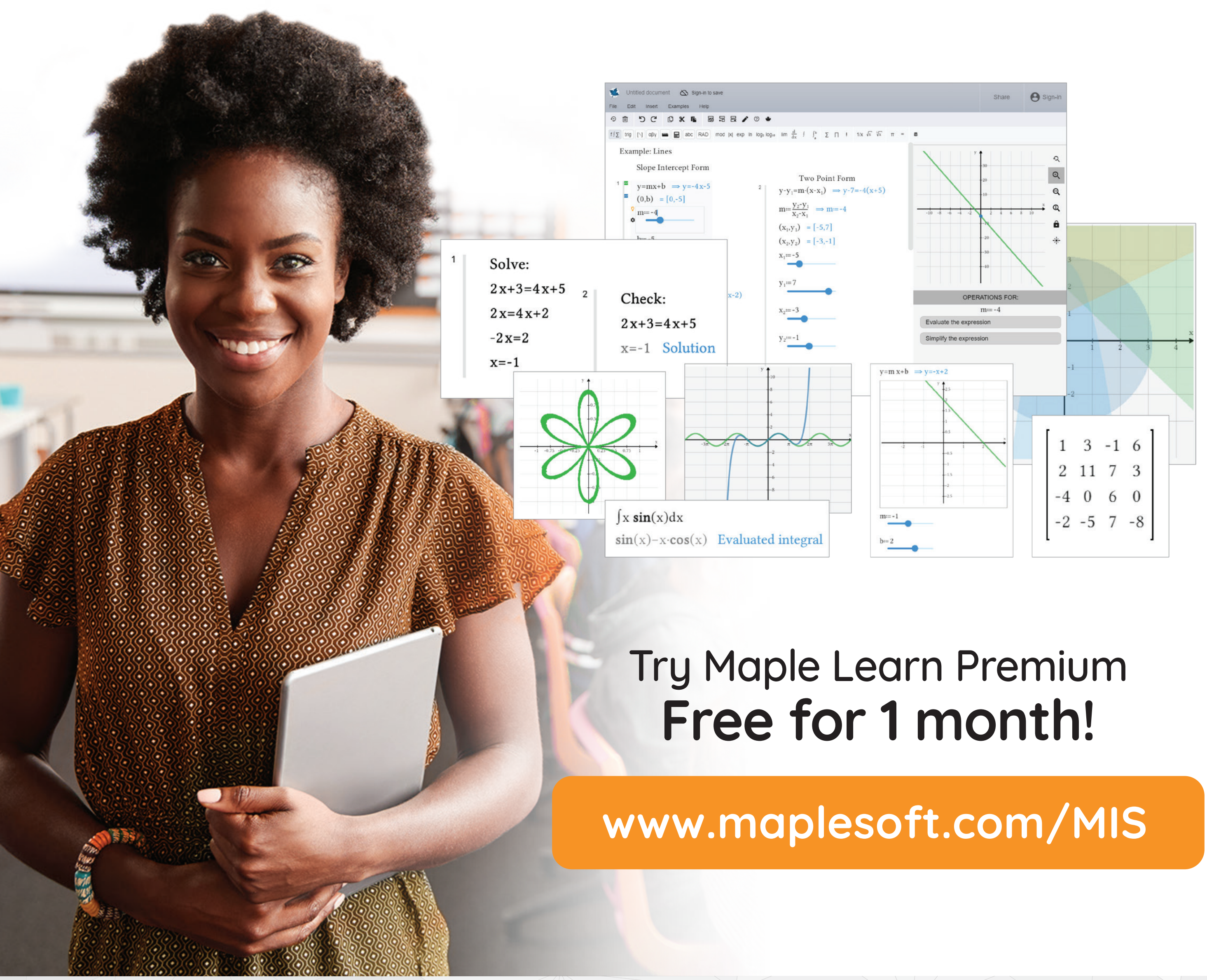

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